SOLUTION

A. Understanding of magnetic fields (1.0 point)

1. Understanding of magnetic field created by a circular coil

A.1	$k = 6.28\times10^{-3}$ mT/mA	0.5 pt

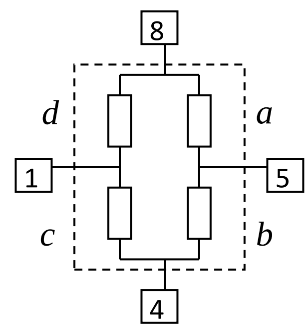
2. Understanding of the Earth's magnetic field

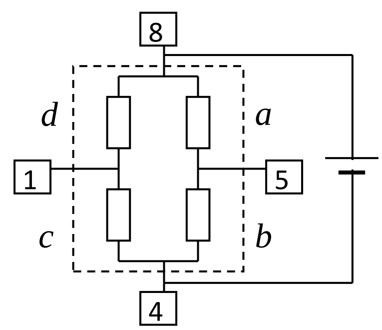
A.2	$B_\beta = B_\mathrm{h}\,\mathbf{cos}\,\beta$	0.5 pt

B. Investigation of the GMR effect using a GMR magnetic sensor (7 points)

2. Determination of resistance of GMR elements

a. ***Resistance of the elements at*** $B = 0$.

<table>
<tr>
<td>B.1</td>
<td>
Diagrams of the experiment and the expressions for calculating the resistance of each element a, b, c and d.
a. Short circuit pins 8 and 4.
$R_{5,84} = m;\ R_{1,84} = n$
$\frac{1}{m} = \frac{1}{a} + \frac{1}{b}$ (1)
$\frac{1}{n} = \frac{1}{c} + \frac{1}{d}$ (2)


b. Connect pins 8 and 4 to a battery.
$\frac{U_{8,5}}{U_{5,4}} = p;\ \frac{U_{8,1}}{U_{1,4}} = q$
$\frac{a}{b} = p$ (3)
$\frac{d}{c} = q$ (4)


Solve the system of equations (1), (2), (3) and (4). Obtain:
$a = m(p+1);\ b = m\left(1+\frac{1}{p}\right)$
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<td>1.25 pt</td>
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	$c = n\left(1+\dfrac{1}{q}\right)$; $d = n\left(q+1\right)$	
B.2	For $B = 0$: $a = 4960\ \Omega$; $b = 4870\ \Omega$; $c = 4950\ \Omega$; $d = 4970\ \Omega$	1.25 pt

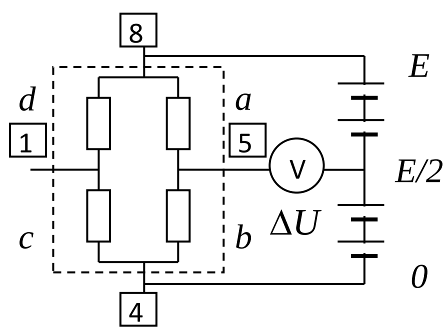
b. ***Resistance of the elements at maximum external magnetic field***

B.3	$a = 4320\ \Omega$; $b = 4870\ \Omega$; $c = 4310\ \Omega$; $d = 4970\ \Omega$	0.5 pt

c. ***Properties of the elements***

B.4	Elements sensitive to the magnetic field are: *a*, *c*	0.25 pt

2. Characteristics of a GMR element

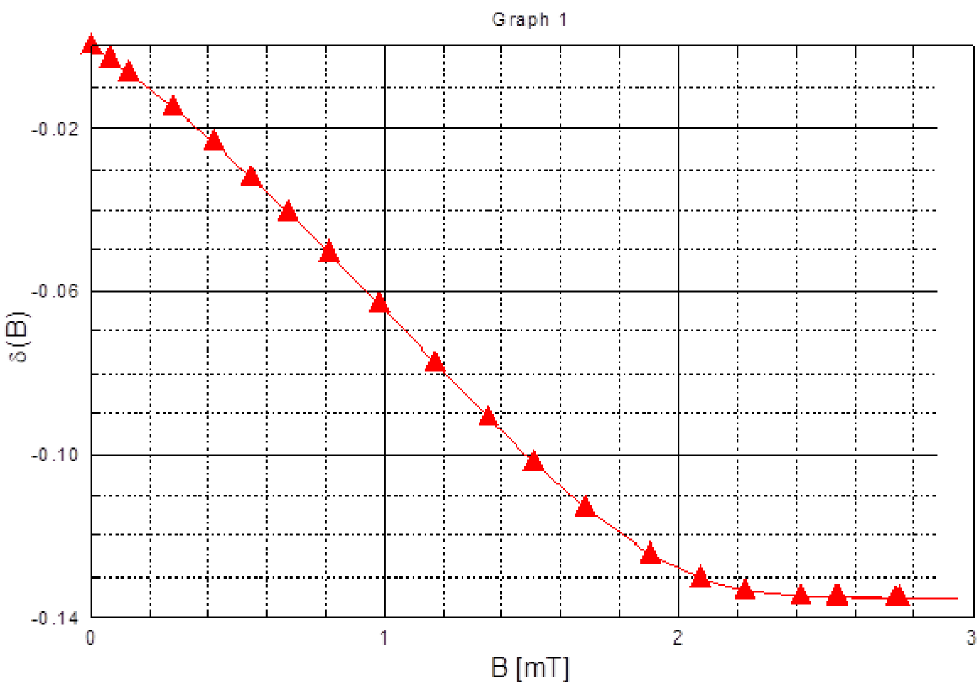
B.5	The name of the chosen element: *a* Diagrams of the experiment and the expressions for calculating $\delta\left(B\right)$. **1.** Method 1: The same as used in B.1 with different values of the current *I* in the circular coil. **2.** Method 2: Connect the sensor to the battery according to the diagram, forming a bridge. The GMR element under consideration is *a*.  If at $I = 0$ the bridge is balanced, then $\Delta U = 0$. Set the current *I* in the coil, the resistance of *a* becomes $R + \Delta R$, then $\Delta U \neq 0$. Because $\Delta U = \dfrac{E \cdot R}{R + R + \Delta R} - \dfrac{E}{2}$, then $\delta\left(B\right) = \dfrac{\Delta R}{R} \approx -\dfrac{\Delta U}{E/4}$. If at $I = 0$, the bridge is unbalanced and the initial voltage is ΔU_0, then $\dfrac{\Delta R}{R} = -\dfrac{\Delta U - \Delta U_0}{E/4}$ and $\delta\left(B\right) = \dfrac{\Delta R}{R} \approx -\dfrac{\Delta U - \Delta U_0}{E/4}$ The voltages are measured relatively to the middle point of the battery.	0.75 pt

	The maximum value of $\Delta R / R$ is about 10%. The error in determining it by using above approximations is less than 1% and can be accepted.	

B.6	Table of $\delta(B)$ corresponding to the values *I* and *B*. $E = 6300\,\text{mV}$	1.25 pt

I (mA)	B (mT)	ΔU (mV)	$\Delta U - \Delta U_0$	$\delta(B)$
0	0	-25.8	0	0
10	0.0628	-21	4.8	-0.00305
20	0.126	-15.7	10.1	-0.00641
45	0.283	-2.1	23.7	-0.01504
67	0.421	11.1	36.9	-0.02343
87	0.546	24.5	50.3	-0.03193
107	0.672	38.1	63.9	-0.04057
129	0.810	54	79.8	-0.05067
156	0.980	74	99.8	-0.06336
186	1.168	96	121.8	-0.07733
215	1.350	117.3	143.1	-0.09085
240	1.507	134.5	160.3	-0.10177
268	1.683	152.6	178.4	-0.11326
303	1.903	170.6	196.4	-0.12469
330	2.072	179.6	205.4	-0.13041
354	2.223	184.1	209.9	-0.13326
384	2.411	186.2	212	-0.13460
405	2.543	186.7	212.5	-0.13492
436	2.738	187.1	212.9	-0.13517
469	2.945	187.2	213	-0.13523

B.7	**Graph 1-** Graph of the relative change of resistance	0.5 pt



B.8	The average slope $\alpha = \frac{\Delta\delta(B)}{\Delta B}$ of the curve $\delta(B)$ $\alpha = -0.067\,\mathrm{mT^{-1}}$	0.25 pts
B.9	The GMR coefficient $\delta = \frac{\Delta R_{\max}}{R(0)} = 13.5\%$	0.25 pts
B.10	The value of the resistances r and R of the GMR element: $r = R_0 - \sqrt{R_0(R_0 - R_B)};\ R = R_0 + \sqrt{R_0(R_0 - R_B)}$ Choose element *a* in B.2 and B.3, then: $r = 3180\Omega;\ R = 6740\Omega;\ \gamma = \frac{r}{R} = 0.47$	0.75 pts

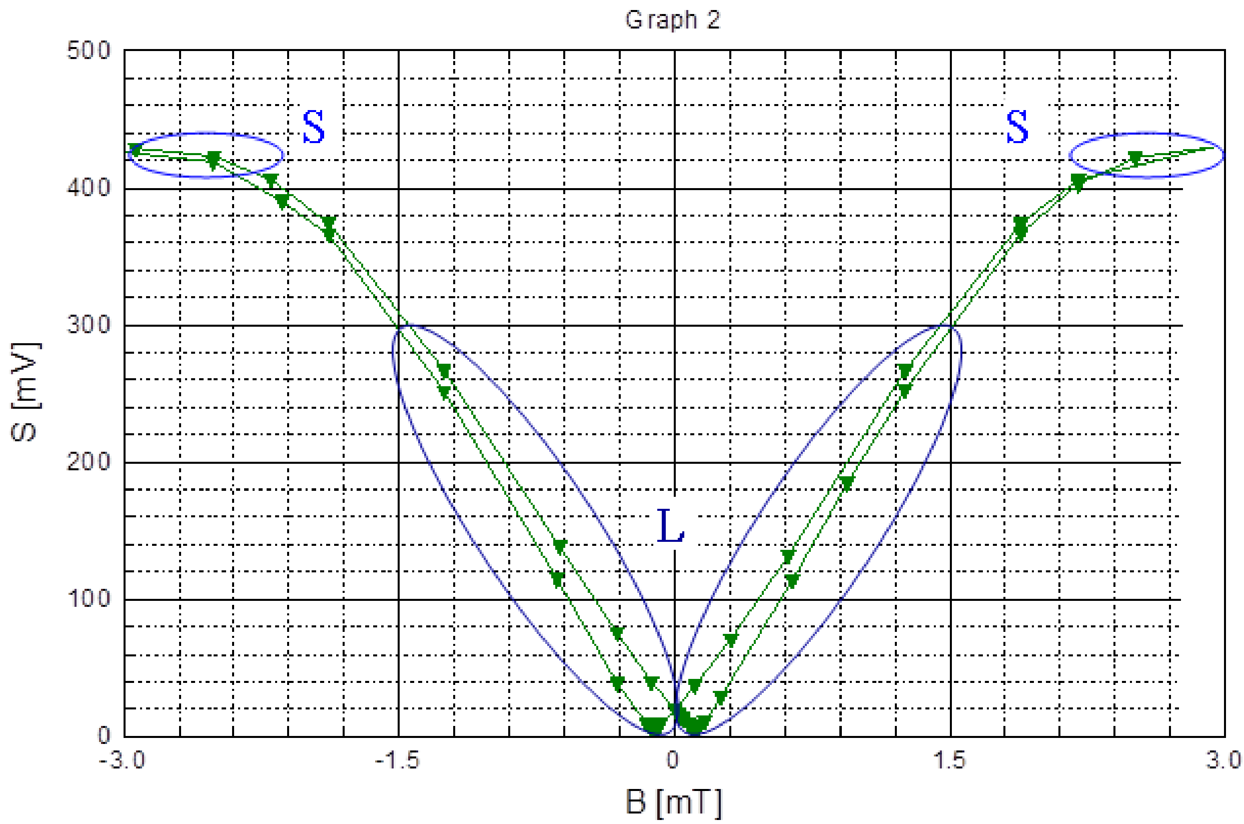
C. Study of GMR magnetic sensor (6 points)

1. Characteristics of sensor output signal

C.1	Table with the values of the output signal S corresponding to the values of the current I and the magnetic field B.	1.0 pts

I	***B***	***S***	***I***	***B***	***S***

C.2	**Graph 2 -** Graph $S(B)$ of the output signal S as a function of the applied magnetic field B.	1.0 pts



C.3	1. Region of saturation in the curve $S(B)$: S 2. Region of linearity in the curve $S(B)$: L $m = 2.0\times 10^2$ mV/mT	0.5 pts
C.4	The coercive field is $B_c = 0.10\,\text{mT}$	0.5 pts

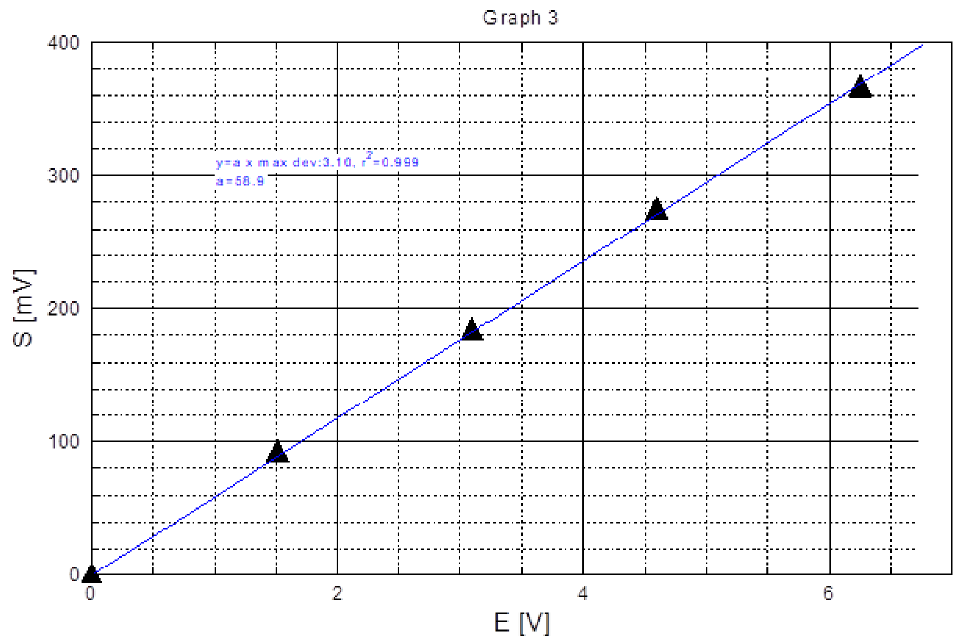
2. Dependence of output signal on the voltage

C.5	Table with the values of S corresponding to the values of E.	0.25 pts

E (V)	S (mV)
0	0

1.51	91.5
3.1	183
4.6	274
6.25	365

C.6	**Graph 3 -** S as a function of E .	0.25 pts



C.7	$\left\|S\right\| = \frac{E}{2} \cdot \left\|\alpha\right\| \cdot B$	0.5 pt

3. Study of effects of a flux concentrator

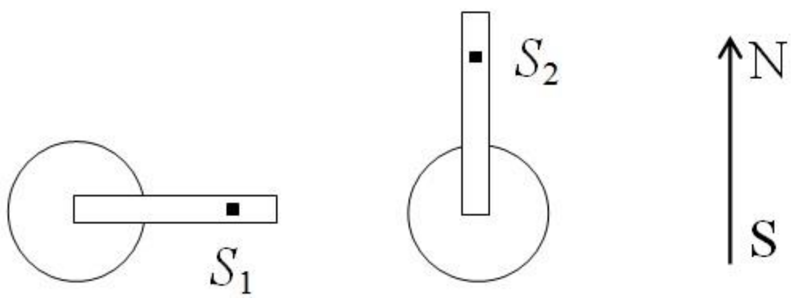
C.8 — 0.25 pt

1. The magnetic field used in this experiment.
Put a cross in the appropriate box

a. The field of the circular coil carrying an electric current	
b. The field of the flat coil carrying an electric current	
c. The field of the plate of permanent magnet	
d. The magnetic field of the Earth	X

2. Diagrams of the experiment and expressions to determine the value of *n*. — 0.75 pt

1. The sensor on the round plate in the horizontal plane.
2. With no flux concentrator



a. Orient the sensor perpendicular to the South-North direction. Note the value S_1.
b. Rotate the sensor along the South-North direction. Note the value S_2.
c. $\Delta S_0 = S_2 - S_1$; $B_0 = |\Delta S_0| / m$.

3.With flux concentrator
For each value of L_1, do the same, to obtain $B = |\Delta S| / m$.

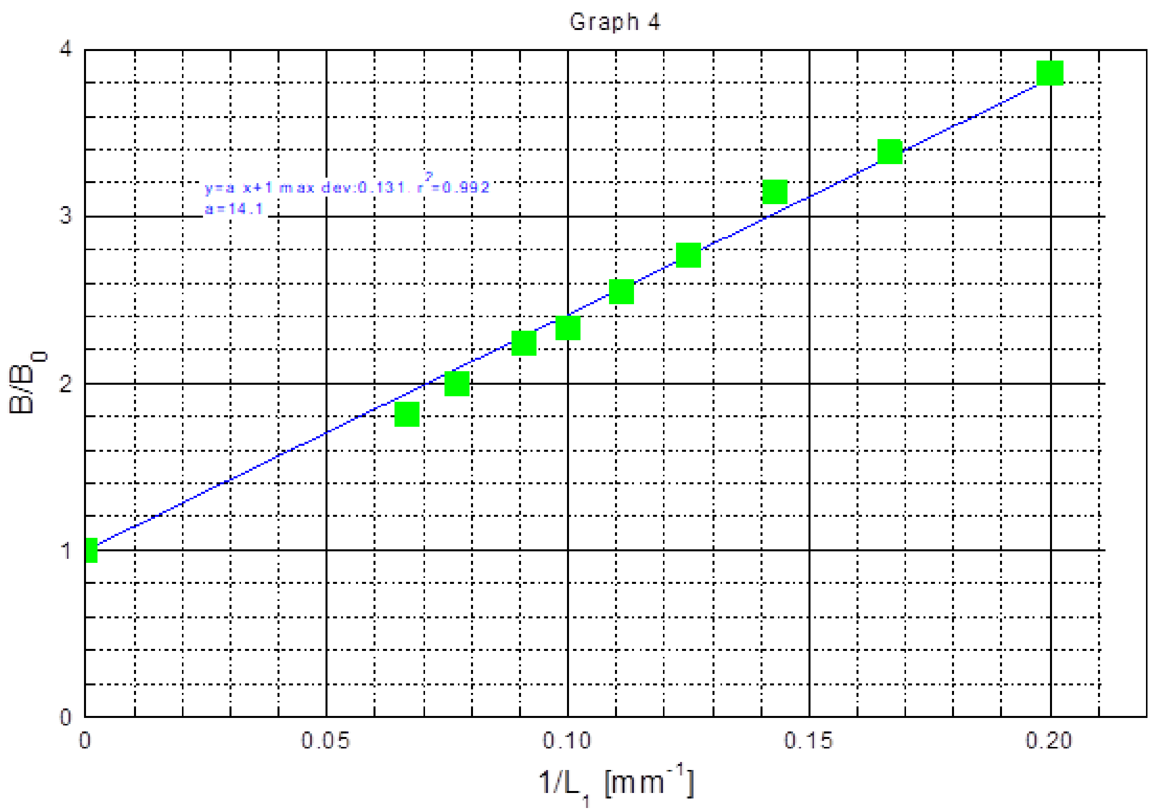
C.9 — 0.5 pt

Table to find B/B_0 for different values of L_1. $B/B_0 = \Delta S / \Delta S_0$
$S_1 = 17\,\text{mV}$; $\Delta S_0 = 21.2 - 17 = 4.2\,\text{mV}$.

L_1 (mm)	S_2 (mV)	$1/L_1$ (mm^{-1})	$\Delta S = S_2 - S_1$	B/B_0
5	33.2	0.200	16.2	3.86
6	31.2	0.167	14.2	3.38
7	30.2	0.143	13.2	3.14

8	28.6	0.125	11.6	2.76
9	27.7	0.111	10.7	2.55
10	26.8	0.100	9.8	2.33
11	26.4	0.0909	9.4	2.24
13	25.4	0.0769	8.4	2.00
15	24.6	0.0667	7.6	1.81
∞	21.2	0.0000	4.2	1.00

C.10	**Graph 4** – Graph of B/B_0 as a function of $1/L_1$. Use the function $\frac{B}{B_0} = nL_2 \cdot \frac{1}{L_1} + 1$. Find $a = nL_2 = 14.1$. Obtain $n = \frac{a}{L_2} = \frac{14.1}{25} = 0.56$.	0.5 pt



D. Applications of GMR magnetic sensors (6 points)

1. Measurements of the Earth's magnetic field

a. Magnitude of the horizontal component of the Earth's magnetic field

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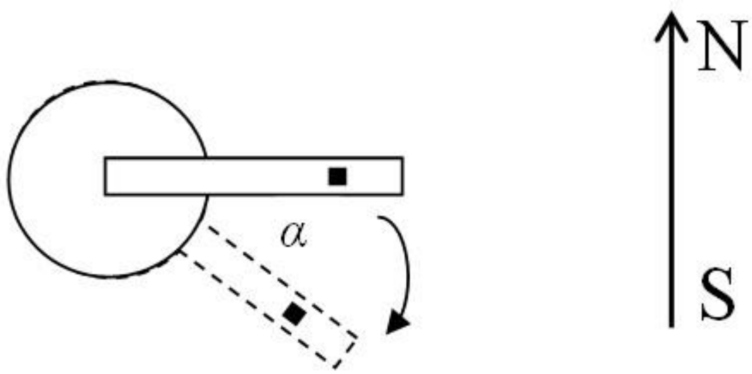
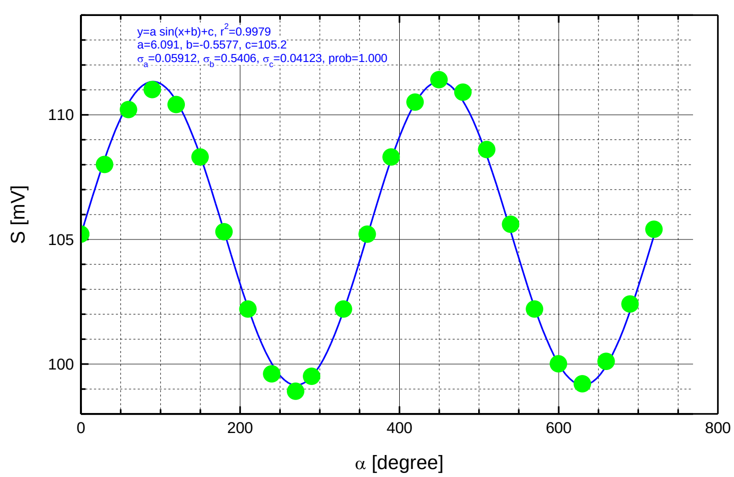
<td>D.1</td>
<td>
Diagrams of the experiment and expressions for calculating B_h .
1. The sensor on the round plate in the horizontal plane. Carry out the biasing.
2. Method 1


a. Set $\alpha = 0$ - the sensor perpendicular to the direction South-North.
b. Rotate the sensor holder, measure $S = f(\alpha)$
c. Fit the curve S to a sine function $S = a\,\mathbf{sin}\,\alpha$.
d. $B_h = a / m$


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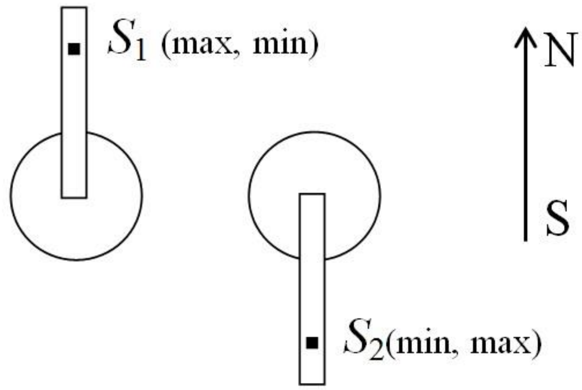
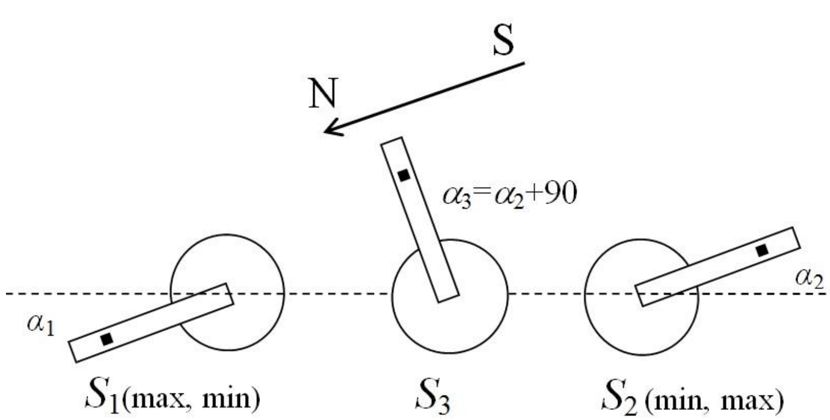
	3. Method 2  a. Orient the sensor along the Earth's magnetic field. Find the direction with the maximum (or minimum) value of S. Note this value S_1 b. Rotate the sensor holder by about 180º. Find the direction with the minimum (or maximum) value of S. Note this value S_2 $B_\mathrm{h} = \frac{\lvert S_1 - S_2 \rvert}{2m}$	

D.2	$B_\mathrm{h} = 0.035\,\mathrm{mT}$.	0.25 pts

b. Magnitude of the Earth's magnetic field and magnetic inclination

D.3	Diagrams of the experiment and expressions for calculating B_Earth and θ. **1.** The sensor on the round plate in the vertical plane containing the South-North direction. Carry out the biasing. **2.** Method 1 	0.75 pts

a. Orient the sensor along the Earth's magnetic field. Find the direction with the maximum (or minimum) value of S. Note this value S_1 and the angle α_1 between the sensor direction and the horizontal.

b. Rotate the sensor holder by about 180°. Find the direction with the minimum (or maximum) value of S. Note this value S_2 and the angle α_2 between the sensor direction and the horizontal.

c. Orient the sensor in the direction midway between α_1 and α_2 with the angle $\alpha_3=\alpha_2+90^\circ$. Note the value S_3.

d. Starting from α_3, rotate the sensor holder, take the values of S corresponding to values of α. Measure $S = f(\alpha)$.

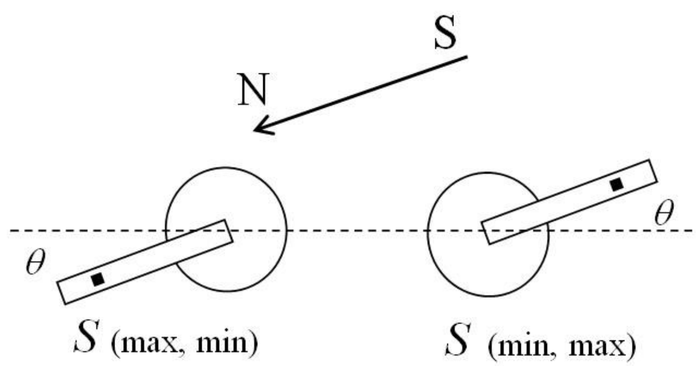
e. $S - S_3 = a\,\mathbf{sin}\,\alpha$. Obtain a from fitting.

f. $B_{\text{Earth}} = a/m$

g. $\theta = \mathbf{Arccos}\dfrac{B_{\text{h}}}{B_{\text{Earth}}}$

3. Method 2

Orient the sensor along the Earth's magnetic field. Find the direction with the maximum (or minimum) value of S. The angle θ between the sensor direction and the horizontal is the magnetic inclination.

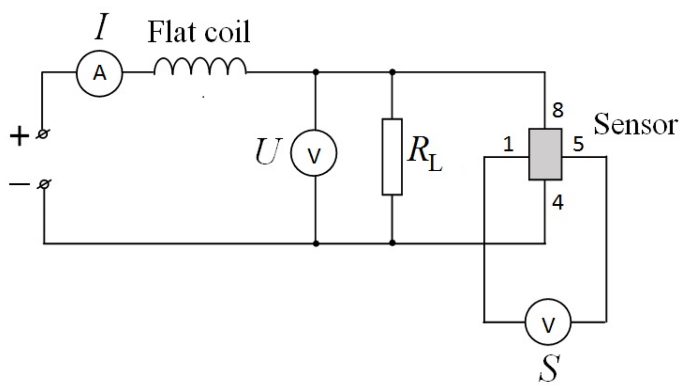


From the obtained θ, $B_{\text{Earth}} = B_{\text{h}} / \mathbf{cos}\,\theta$.

This method may have systematic errors due to the relative misalignment of the sensor to the sensor holder. To eliminate this error, rotate the round plate together with the sensor holder by 180° about a horizontal axis along the South-North direction. Repeat the measurement. The magnetic inclination is the mean value of the

	two obtained angles.	
D4	$B_{\text{Earth}} = 0.041\text{mT}$ $\theta = 31^{\text{o}}$	0.5 pts

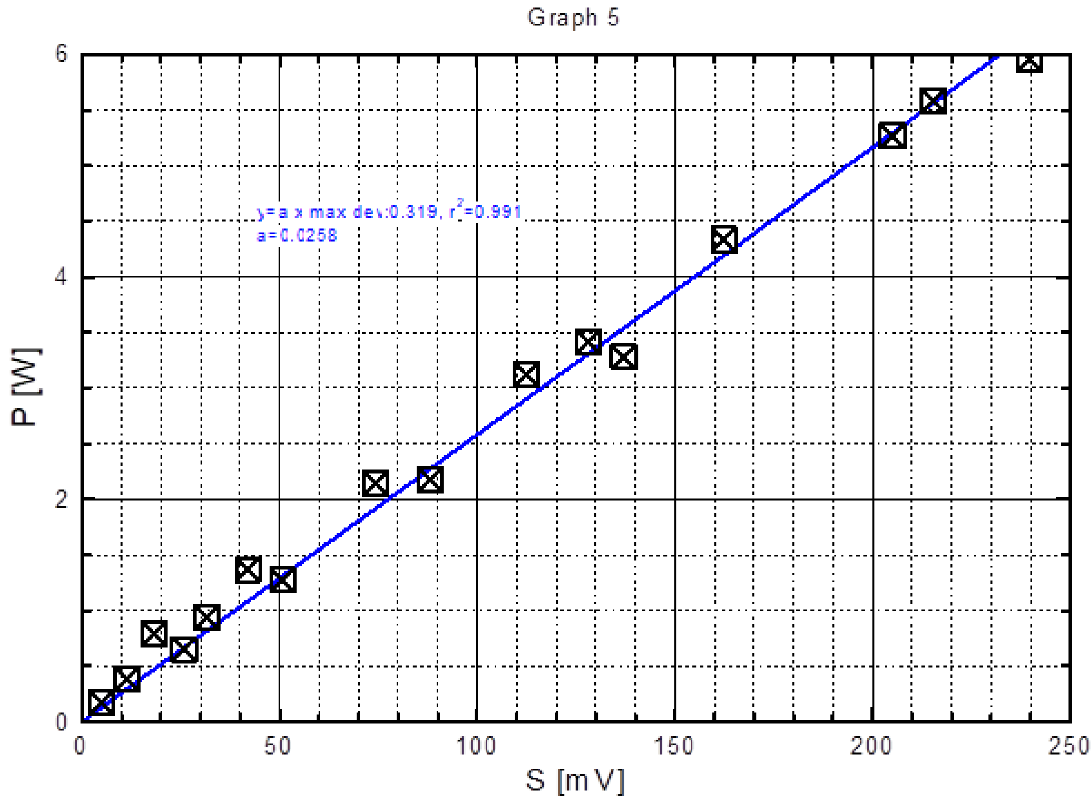
2. DC wattmeter

D.5	Diagram of the wattmeter circuit together with the load and the multimeters. 	0.5 pt

D.6	Table with the values of the sensor output signal *S* corresponding to the values of *I* and *U*, and of $P = I \cdot U$.	0.75

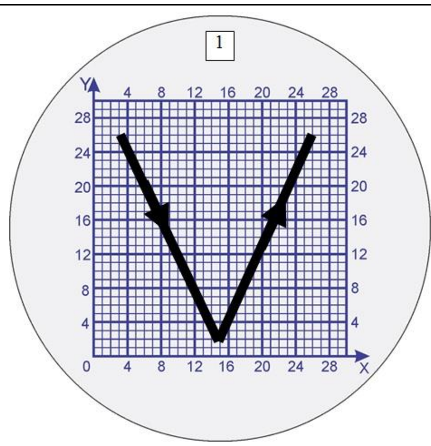
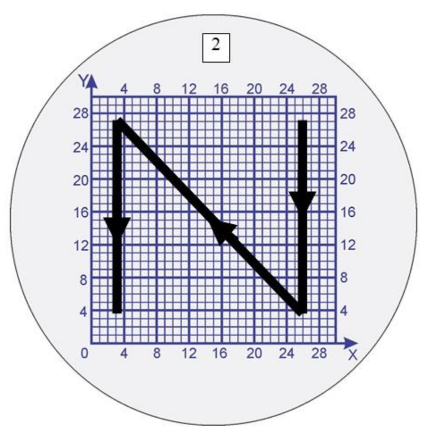
I (A)	*U* (V)	*P* (W)	*S* (mV)
0.30	2.64	0.792	18.3
0.35	3.9	1.365	42
0.40	5.37	2.15	74.3
0.45	6.94	3.12	112.4
0.50	8.67	4.34	162.4
0.543	10.29	5.59	215.4
0.20	0.89	0.178	4.9
0.25	1.53	0.382	11.5
0.50	1.3	0.65	25.8
0.60	2.13	1.28	50.7
0.70	3.1	2.17	88.1
0.80	4.1	3.28	137
0.97	6.11	5.92	253
0.30	3.13	0.939	31.4
0.442	7.74	3.42	128

D.7	**Graph 5 -** Calibration curve of the wattmeter $P = f\left(S\right)$.	0.5 pt



D.8	The function: $P = \kappa S$ The coeficient: $\kappa = 0.026$ W/mV	0.25 pt

b. Detection of buried electrical circuits

D.9	 	2.0 pt