17th Asian Physics Olympiad 1-9 May 2016

Theoretical Question – T2

# The Expanding Universe

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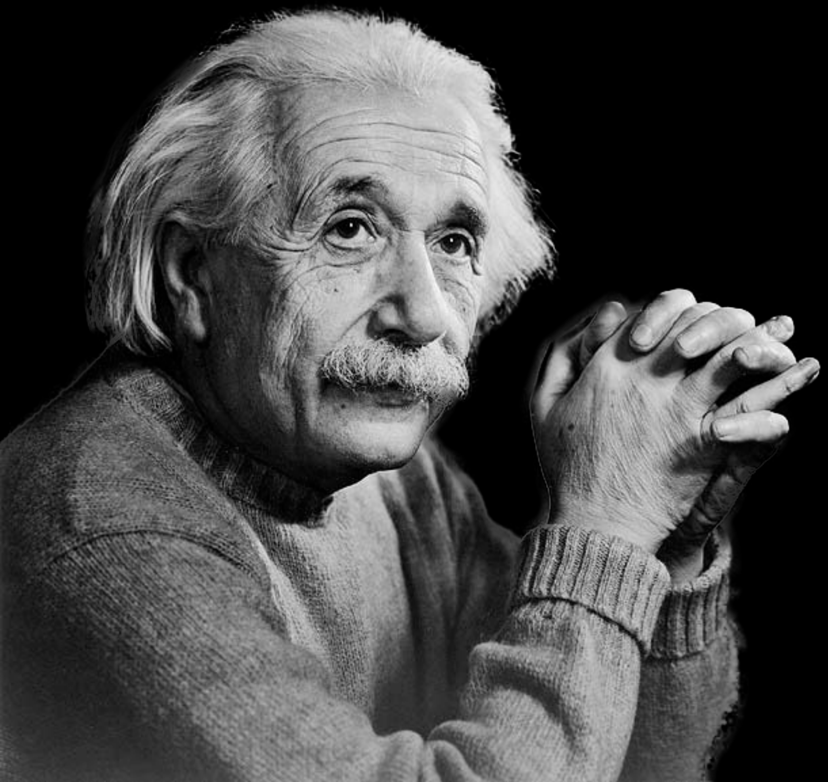
The cosmological principle:

On large scales, the universe is approximately homogeneous and isotropic.



Einstein’s theory of gravity:

Space tells matter how to move

$$\nabla^{\mu} T_{\mu\nu} = 0$$

Matter tells space how to curve

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Here we shall understand the expansion of the universe

using **Newton's gravity** theory,

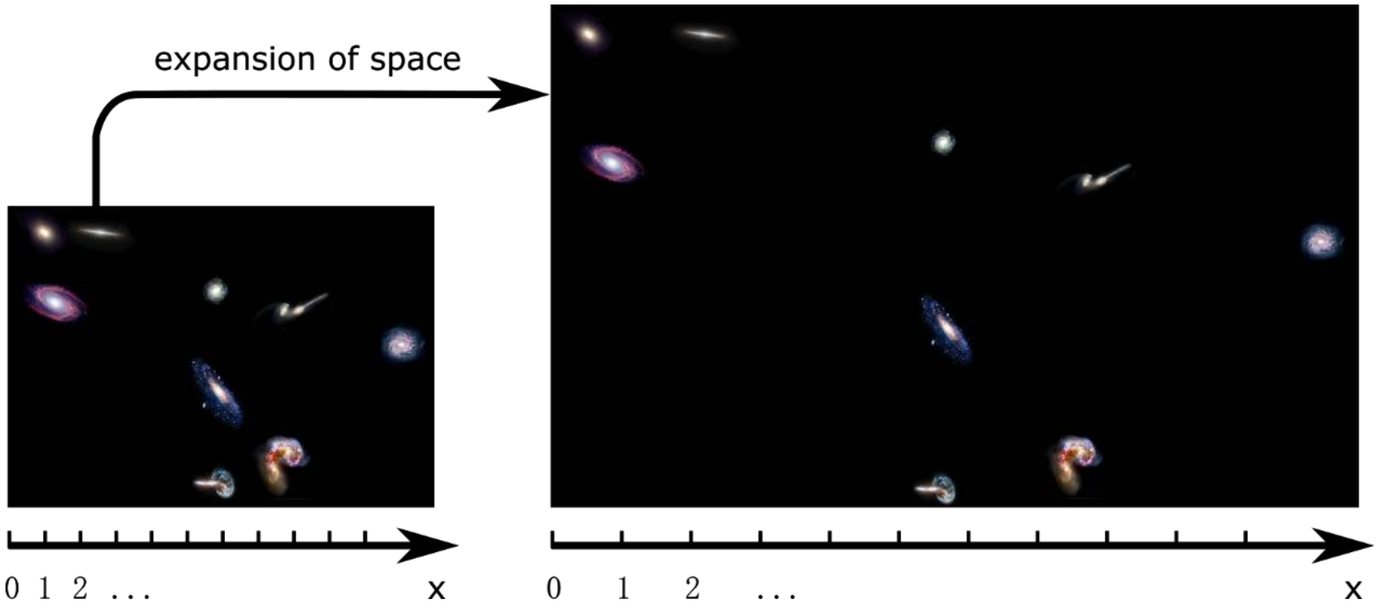
combined with **thermodynamics of gas**,

and a bit of **special relativity** and **quantum mechanics**.

How to understand the Universe expansion?

In Newton's view:

Objects are thrown outwards at the big bang.




$$\Delta r_p = a(t)\Delta r$$

physical comoving

We shall test 3 topics of physics with the Universe expansion:

- Space tells matter how to move:

  How matter energy density changes with $a(t)$

- Matter tells space how to curve:

  How $a(t)$ is determined by the matter energy density.

- Distance measurements:

  How a distant “standard candle” looks like to us.

Topic I: Space tells matter how to move.

Given the equation-of-state of matter $X : p_X = p_X(\rho_X)$,

what is $\mathrm{d}\rho_X/\mathrm{d}t$ in terms of $\rho_X$, $p_X$, $a(t)$ and $\mathrm{d}a(t)/\mathrm{d}t$?

A: Example of non-relativistic matter

B & C: Example of ultra-relativistic matter

D: The general situation

Idea: 1st law of thermodynamics (or energy conservation)

$$\mathrm{d}E_\mathrm{X} = -p_\mathrm{X}\mathrm{d}V_\mathrm{p}$$ (entropy keeps the same)

$$E_\mathrm{X} = \rho_\mathrm{X} V_\mathrm{p}$$

$$V_\mathrm{p} = a^3 V$$

$$\mathrm{d}E_\mathrm{X} = a^3 V \mathrm{d}\rho_\mathrm{X} + 3\rho_\mathrm{X} a^2 V \mathrm{d}a$$

$$\mathrm{d}V_\mathrm{p} = 3a^2 V \mathrm{d}a$$

$$\mathrm{d}\rho_\mathrm{X} + 3\left(\frac{\mathrm{d}a}{a}\right)(\rho_\mathrm{X} + p_\mathrm{X}) = 0$$

$$\dot{\rho}_\mathrm{X} + 3\left(\frac{\dot{a}}{a}\right)(\rho_\mathrm{X} + p_\mathrm{X}) = 0$$

Topic II (F & G) : Matter tells space how to curve

Given non-relativistic matter ($p = 0$),

homogeneous & isotropic distribution,

find relation between $\mathrm{d}a(t)/\mathrm{d}t$, $a(t)$ and $\rho(t)$.

If no gravity: matter flies away freely

relative to “the center of the universe”,

thus $\mathrm{d}a(t)/\mathrm{d}t = \mathrm{const}$ (Newton’s first law)

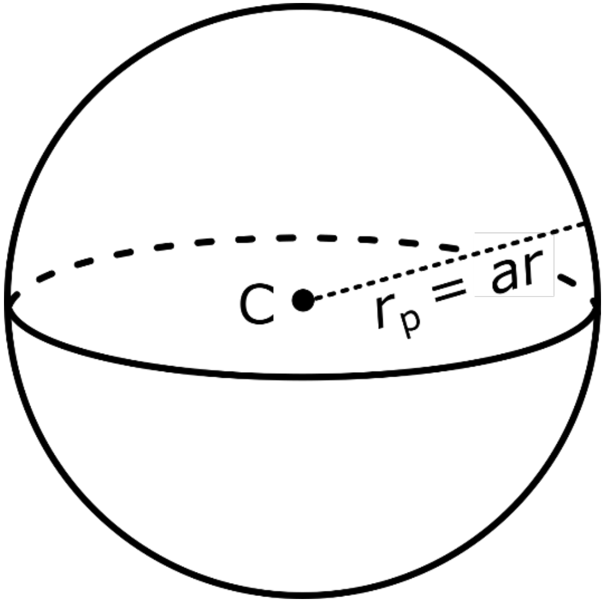
With gravity:

$$E = \frac{1}{2} m (\dot{r}_p)^2 - \frac{GMm}{r_p} = \text{const}$$

$$M = \frac{4\pi}{3} {r_p}^3 \rho$$

$$r_p = a(t) r$$

$$\frac{2E}{mr^2} = \dot{a}^2 - \frac{8\pi G}{3} \rho a^2$$




Note: the conventional form:

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{k}{a^2} = \frac{8\pi G}{3} \rho$$

# Topic III (E): Distance measurements

Distance?

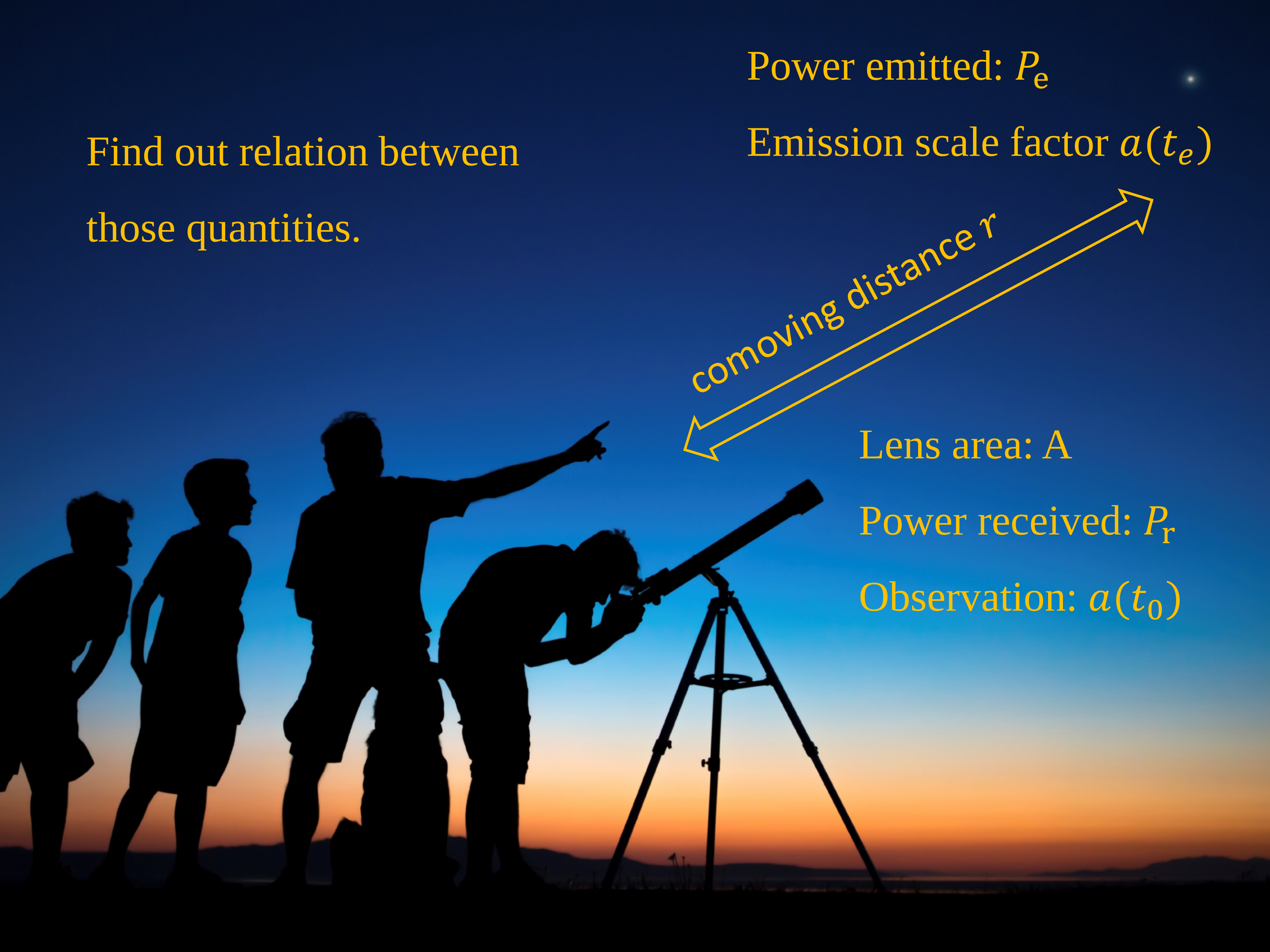
Standard candle: Assume

the same absolute luminosity,

apparent luminosity

↓

distance


Power emitted: $P_e$
Emission scale factor $a(t_e)$
Find out relation between
those quantities.
comoving distance $r$
Lens area: A
Power received: $P_r$
Observation: $a(t_0)$

3 things to consider:

(a) Area ratio: $A/(4\pi a^2(t_0)r^2)$

(b) Energy per photon: $a(t_e)/a(t_0)$

(c) Photon interval: $a(t_e)/a(t_0)$

$$P_r = \frac{A\, a^2(t_e)}{4\pi a^4(t_0) r^2} \times P_e$$

All answers can be verified in General Relativity.

But that’s beyond the scope of this question.


Thank you!