Question 3

Figure 1 shows a Fabry-Perot (F-P) etalon, in which air pressure is tunable. The F-P etalon consists of two glass plates with high-reflectivity inner surfaces. The two plates form a cavity in which light can be reflected back and forth. The outer surfaces of the plates are generally not parallel to the inner ones and do not affect the back-and-forth reflection. The air density in the etalon can be controlled. Light from a Sodium lamp is collimated by the lens L1 and then passes through the F-P etalon. The transmitivity of the etalon is given by $T = \frac{1}{1 + F\sin^2(\delta/2)}$, where $F = \frac{4R}{(1-R)^2}$, R is the reflectivity of the inner surfaces, $\delta = \frac{4\pi nt\cos\theta}{\lambda}$ is the phase shift of two neighboring rays, n is the refractive index of the gas, t is the spacing of inner surfaces, θ is the incident angle, and λ is the light wavelength.

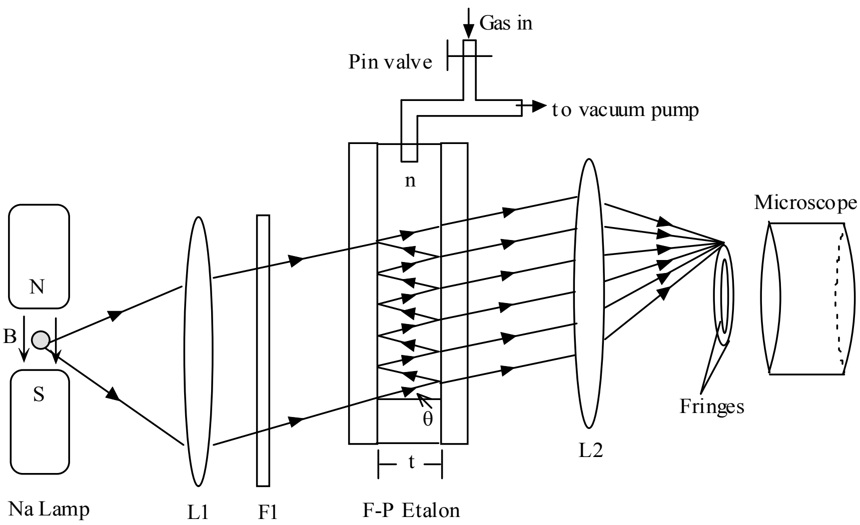

Figure 1

The Sodium lamp emits D1 ($\lambda = 589.6nm$) and D2 ($589nm$) spectral lines and is located in a tunable uniform magnetic field. For simplicity, an optical filter F1 is assumed to only allow the D1 line to pass through. The D1 line is then collimated to the F-P etalon by the lens L1. Circular interference fringes will be present on the focal plane of the lens L2 with a focal length f=30cm. Different fringes have the different incident angle θ. A microscope is used to observe the fringes. We take the reflectivity R= 90% and the inner-surface spacing t=1cm.

Some physical constants: $h = 6.626\times10^{-34} J\cdot s$, $e = 1.6\times10^{-19} C$, $m_e = 9.1\times10^{-31} kg$, $c = 3.0\times10^{8} ms^{-1}$.

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（a） **（3points）** The D1 line ($\lambda = 589.6nm$) is collimated to the F-P etalon. For the vacuum case (n=1.0), please calculate (i) interference orders m_i, (ii) incidence angle θ_i and (iii) diameter D_i for the first three（i=1，2，3）fringes from the center of the ring patterns on the focal plane.

（b）**（3 points）** As shown in Fig. 2, the width ε of the spectral line is defined as the full width of half maximum (FWHM) of light transmitivity T regarding the phase shift δ. The resolution of the F-P etalon is defined as follows: for two wavelengths λ and $\lambda + \Delta\lambda$, when the central phase difference $\Delta\delta$ of both spectral lines is larger than ε, they are thought to be resolvable; then the etalon resolution is $\lambda / \Delta\lambda$ when $\Delta\delta = \varepsilon$. For the vacuum case, the D1 line ($\lambda = 589.6nm$), and because of the incident angle $\theta \approx 0$, take $\cos\theta \approx 1.0$, please calculate：

（i）the width ε of the spectral line.

（ii）the resolution $\lambda/\Delta\lambda$ of the etalon.

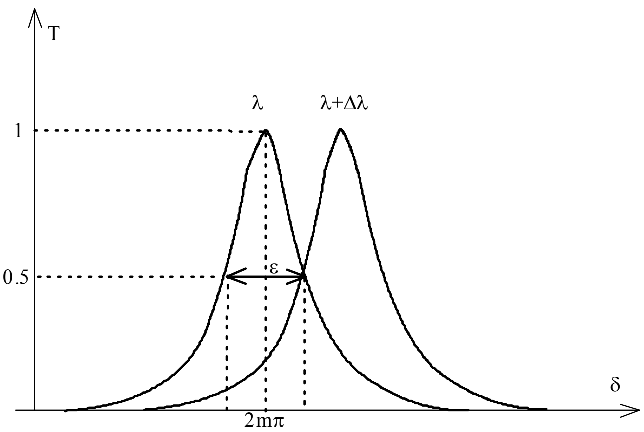

Figure 2

（c）**（1 point）** As shown in Fig. 1, the initial air pressure is zero. By slowly tuning the pin valve, air is gradually injected into the F-P etalon and finally the air pressure reaches the standard atmospheric pressure. On the same time, ten new fringes are observed to produce from the center of the ring patterns on the focal plane. Based on this phenomenon, calculate the refractive index of air n_{air} at the standard atmospheric pressure.

（d） **(2 points)** Energy levels splitting of Sodium atoms occurs when they are placed in a magnetic field. This is called as the Zeeman effect. The energy shift given by $\Delta E = m_j g_k \mu_B B$, where the quantum number m_j can be J，J-1，…，-J+1，-J，J is the total angular quantum number,

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g_k is the Landé factor, $\mu_B = \frac{he}{4\pi m_e}$ is Bohr magneton，h is the Plank constant，e is the electron charge，m_e is the electron mass, B is the magnetic field. As shown in Fig. 3, the D1 spectral line is emitted when Sodium atoms jump from the energy level $^2P_{1/2}$ down to $^2S_{1/2}$. We have $J = \frac{1}{2}$ for both $^2P_{1/2}$ and $^2S_{1/2}$. Therefore, in the magnetic field, each energy level will be split into two levels. We define the energy gap of two splitting levels as ΔE_1 for $^2P_{1/2}$ and ΔE_2 for $^2S_{1/2}$ respectively ($\Delta E_1 < \Delta E_2$). As a result, the D1 line is split into 4 spectral lines (a, b, c, and d), as showed in Fig. 3. Please write down the expression of the frequency (ν) of four lines a, b, c, and d.

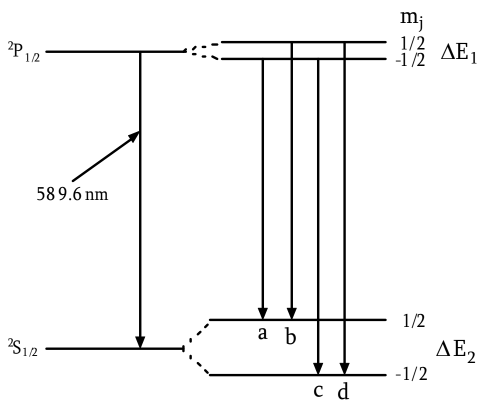

Figure 3

(e) **(3 points)** As shown in Fig. 4, when the magnetic field is turned on, each fringe of the D1 line will split into four sub-fringes (1, 2, 3, and 4). The diameter of the four sub-fringes near the center is measured as D_1, D_2, D_3, and D_4. Please give the expression of the splitting energy gap ΔE_1 of $^2P_{1/2}$ and ΔE_2 of $^2S_{1/2}$.

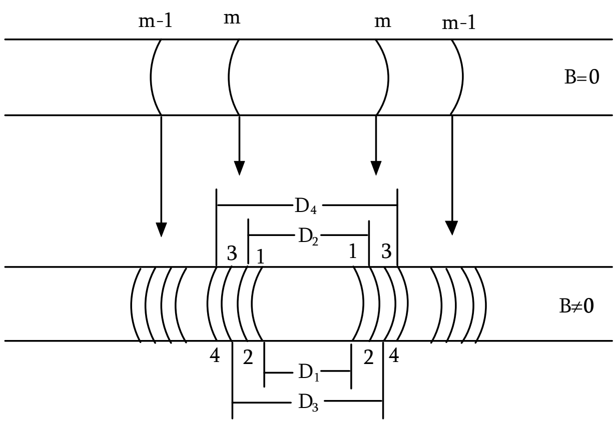

Figure 4

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（f）（**3 points**）For the magnetic field B=0.1T，the diameter of four sub-fringes is measured as：$D_1 = 3.88mm$， $D_2 = 4.05mm$， $D_3 = 4.35mm$， and $D_4 = 4.51mm$. Please calculate the Landé factor g_{k1} of $^2P_{1/2}$ and g_{k2} of $^2S_{1/2}$.

（g） **(2 points)** The magnetic field on the sun can be determined by measuring the Zeeman effect of the Sodium D1 line on some special regions of the sun. One observes that, in the four split lines, the wavelength difference between the shortest and longest wavelength is 0.012nm by a solar spectrograph. What is the magnetic field B in this region of the sun?

(h)（**3 points**）A Light- Emitting Diode (LED) source with a central wavelength $\lambda = 650nm$ and spectral width $\Delta\lambda = 20nm$ is normally incident ($\theta = 0$) into the F-P etalon shown in Fig. 1. For the vacuum case, find (i) the number of lines in transmitted spectrum and (ii) the frequency width $\Delta\nu$ of each line?