Experiment. To see invisible! (20 points)

Introduction

Many substances exhibit optical anisotropy resulting in that the refractive index turns out dependent on the direction of light propagation and its polarization. Optical anisotropy can occur even in isotropic media in the presence of mechanical stresses, nonuniform heating or application of external electric fields. The direction in which the light propagates without the birefringentis called the optic axis of a crystal.

Consider a traditional optic scheme of experiments for studying the optical anisotropy (see Fig. 1), which is to be used in this experimental problem.

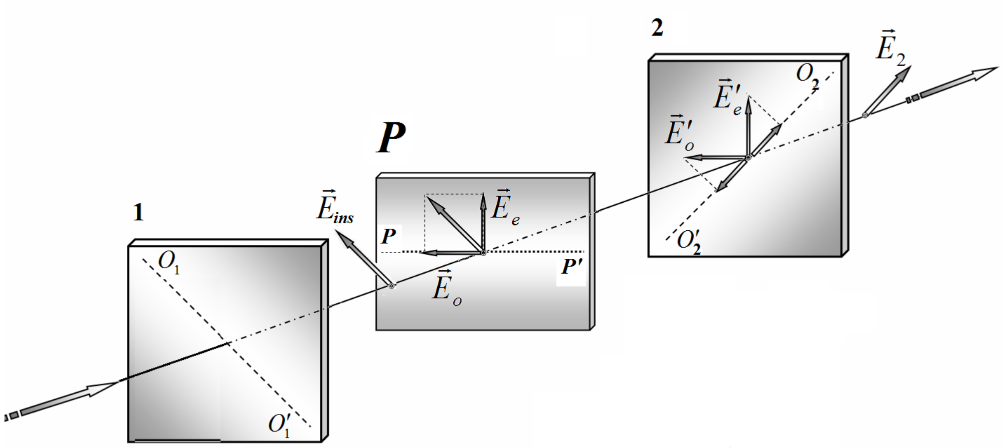


Fig. 1. Optic scheme of an experiment for studying the optical anisotropy.

Let a light beamfall onto polarizer 1 whose transmission plane intersects its ownplane along the straight line $O_1O_1^{'}$. After passing polarizer 1 the light beam becomes linearly polarized and its electric field strength vector $\vec{E}_0$ oscillatesexactly in the transmition plane of polarizer 1. Then, the light beam falls onto the anisotropic plate P oriented such that its optical axisPP'lies in the plate plane to makethe angle 45° with the transmission plane of polarizer 1. Two kinds of light waves are then generated in the plate P: ordinary $\vec{E}_o$, polarized perpendicular to the optical axis of the plate, and extraordinary $\vec{E}_e$, polarized along the optical axis of the plate. Refractive indices for these two waves are different and their difference is denoted as $\Delta n = n_o - n_e$. This results in the appearanceof the phase difference $\Delta\varphi = 2\pi h\Delta n/\lambda$($h$ being the plate thickness, λ being the wavelength of the incident light in vacuum) between the two waves on leaving the plate. Therefore, the polarization of the outgoing light beam changes to be elliptically polarized. The light beam then falls onto polarizer 2, whosetransmission plane $O_2O_2^{'}$is perpendicular to the transmission plane of polarizer 1.

A simple derivation shows that the intensity of the light beam transmitted through the plate P and polarizer 2 is determined as

$$I_2 = kI_0 \sin^2\frac{\Delta\varphi}{2}, \tag{1}$$

whereI_0stands for the light intensity falling onto the plate,k denotes the light transmittance coefficient of the plate P and polarizer 2, and $\Delta\varphi$designates the phase difference between the ordinary and extraordinary waves after passing the plate P.

In this experiment do not evaluate errors unlessasked to do so!
The description of the equipment in Appendix A

Part 1.Qualitative observations!(3.5 points)

Part 1.1.Polarizers(0.8 points)

1.1	Find the orientation(i.e. which of the diagonals) of the transmission plane of polarizer 1 and polarizer 2. Show these planes in the figure in the Answer Sheet. (0.8 points)

Part 1.2.Rulers(1.0 points)

In this part of the experiment uselight emitting diodes(LED) as a light source.

Fix LED on a stand and connect it to its power supply. Set up both polarizers by their face sides (indicated bynumbers 1 and 2) pointing towards the light source. Make sure that the polarizers are crossed, i.e. the light beamcannot pass through them. Block the first polarizer by a sheet of white paperplacing it on the face side as shown in Fig.1B in Appendix B.

Place the plastic ruler1between the polarizers. You can move the ruler with your hands.

1.2.1	Find the possible directions of the optical axis in thecenter of the plastic ruler. Show these directions in the figure on the Answer Sheet. (0.4 points)
1.2.2	Determine approximately at what distance along the ruler 1 and along the two rulersstacked together, the phase difference for the blue light changes to 2π. (0.6 points)

Part 1.3.Strip(0.8 points)

1.3.1	Find the possible directions of the optical axis of the strip. Show qualitatively them in the figure in the Answer Sheet. (0.4 points)

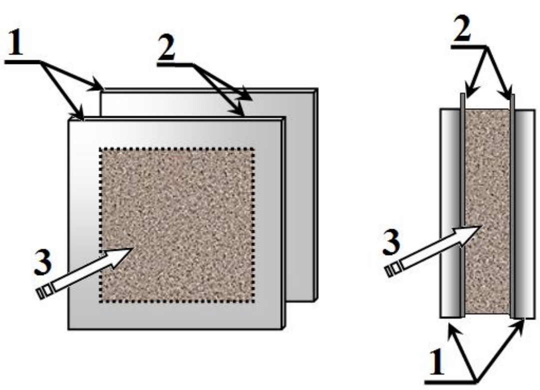
Using the clamps fix a long flexible plastic strip on the screen so that the strip edges coincide with the screenedges. The stripshould be curved (see Fig. 3B). Place the screen with the strip between the polarizers. Shifting the screen, observe the color change of the strip. Measure *x* coordinates of strip points on the screen scale, use the left edge of the screenholder as an origin as shown in Figure 3B.

Hereinafter coordinates are measured by the scale in the screen. As a pointer, use the left edge of the holder, which indicated in Fig. 3B by the arrow!

1.3.2	Measure the coordinates of the middle points of two dark bands, the left $x_L$and the right x_R, visible on the strip. (0.4 points)

Part 1.4.Liquid crystal cell(0.9 points)

Liquid crystal (LC) is a state of matter that is intermediate between the crystalline solid and the amorphous liquid. The orientation of its molecules can be easily aligned and controlled by applying an electrical field. The LC cell exhibits the optical anisotropy phenomenon with two principal refractive indices. The magnitude of this effect depends on the applied AC voltage. The Liquid Crystal Cell (LCC) is composed of two glass plates 1 whose inner surface is coated with a transparent conductive layer 2. Between the plates there is a thin (approximately 10 microns) layer of the solution 3 which is in a liquid crystal state. Leads are soldered to the plates for connecting to the AC power supply.



Place the liquid crystal cell (LCC) between the polarizers. Plug it into its power supply. Varying the voltage across the LCC observe the changing colors of the light transmitted through it.

1.4.1	Find the possible directions of the optical axis of the LCC at zero and maximum voltageapplied across it. Show these directions in the figure on the Answer Sheet. *Z*-axis is directed vertically.(0.6 points)

1.4.2	Measure the voltage U_{cr} across the cell at which abrupt,with respect to voltage, reorientation of liquid crystal molecules by 90° occur.Make sure that multimeter is on AC voltage mode.(0.3 points).

Part 2.Measure!(16.5 points)

Disconnect the LED from the power supply and remove it. Remove the sheet of white paper.
In this part of the work use laser as a light source, make sure you connect it to its power supply!

Fixthe laser, polarizer 1, the screen with the slit and a photodetector (a photodiode) in the holders. Adjust the setting so that the laser beam passes through the polarizer andthe slit of the screen to fall strictly onto the photodiode. Using the screw 5c for the beam width adjustment, make sure that the spot size on the photodetectoris about 5-6 mm.

The laser emits a linearly polarized light. With the ring 5a designed for the laser reorientation,make sure that the laser beam almost completely passes through the first polarizer and the major axis of the elliptically-shaped spot is vertical. In the following,the orientations of the laser and the photodetectormust be fixed with the screws 5d and15c. Set up polarizer 2. Make sure that the polarizers are crossed. Fig. 4B shows the whole setup with the screen mounted.

Part2.1.Investigating a photodiode(3.2 points)

For measuringthe light intensity,of use is the photodiode EMF which is a rather complicated function of the incident light intensity itself. Therefore, for measuring the light intensity the circuit shown in Fig. 2 is used. The DC voltage measured by a multimeter depends on the incident light intensity and the resistance of a resistor. **The main objective is to choose such an optimum value of the resistance thatthe voltage across the resistor is to be proportional to the intensity of the light incident on the photodiode.**

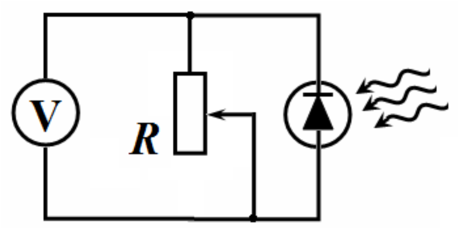


Fig. 2. Circuit for measuring the photodiode emf.

For measurements in this part, remove the second polarizer and the screen from the optical bench.
The filters that attenuate the beam intensity should be necessarilyfixed with the clamps on the back side of the polarizer as shown in Fig. 5B.
The maximum values of the measured voltages must be at least 300 mV.

Using a multimeter, you can measure the resistance of the resistor and the voltage across it (of course, you have to properlyadjust the register of the multimeter). Add, in an appropriate place,the switch provided so that you could measure both the resistance and the voltage with the single multimeter, i.e. without disconnecting the circuit by just shorting/unshorting the switch and the multimeter register adjustment.

2.1.1	Draw a circuit with the switch installed for measuring of the voltage across the resistor and its resistance. (0.2 points)
2.1.2	Measure the voltage across the resistor as a function of its resistance for two values of the incident light intensity: maximum (with the number of filters $n = 0$) and the minimum (with the number of filters $n = 5$). Plot the corresponding graphs in the same figure. Specify the range of the resistance for which the difference between the voltages is maximal. (1.0 point)
2.1.3	Measure the voltage U across the resistor as a function of the number of filters $n = 0,1,2,3,4,5$, attenuating the intensity of the incident light on the photodiode. Measurements must be carried out at three fixed values of the resistance, approximately equal to $R = 30\ kOhm$, $R = 20\ kOhm$ and $R = 10\ kOhm$.Plot the corresponding graphs in the same figurechoosing the scale such that it would be possible to verify whether the voltage across the resistor depends linearly on the intensity of the incident light registered by the photodiode. From the three above mentioned values of the resistance choose an optimum one R_{opt} at which further measurements of the light intensity should be made. (1.0 point)
2.1.4	Using this data obtained, calculate the transmittance of the filter $\gamma = I_{tr}/I_{inc}$ and evaluate its

	error,withI_{tr} being the intensity of the transmitted light, and I_{inc} being the intensity of the incident light. You can make additional measurements if necessary. (1.0 point)

All subsequent measurements must be made at the optimal value of the resistor chosen!
It is assumed in what follows that the light intensityin relative units is equal to the voltage across the resistor in mV.

Part 2.2 Light transmission through plastic rulers(5.4 points)

Place the plastic ruler between the polarizers. You can move the ruler with your hands. Then, secure it with the clamps on the screen with a slit (see Fig. 2B). The lower edge of the ruler should coincide with a line drawn on the screen, and its scale should be at the top. Make sure that both of the rulers provided demonstrate the birefringence phenomenon. Observe a picture emerging when you put one ruler on the top of the other so that light is to pass through both of them.

In this part use an optical scheme shown in Fig. 4B. Make sure that the rulersare fixed on the screen at the position described in Part 1.2.

2.2.1	Measure the intensity of the transmitted light (in mV) as a functionof the coordinate$x$ of the point of the light incidence on the ruler in the range from 0 to 10 cm.. Measurements must be carried out for each ruler provided and for the two rulers stacked together. In each case, measure the maximum value of the voltage. Plot the corresponding graphs in the same figure. (2.0 points)
2.2.2	For each of the two rules calculate the values of the phase shift $\Delta\varphi$ between the ordinary and extraordinary waves in the range of x from 0 to 7 cm.Plotthe correspondinggraphs$\Delta\varphi(x)$.Put down the formula you have used for calculations. (1.2 points) ***Note that the phase difference cannot unambiguously be determined from formula (1),additional physical assumptions should be applied to determine it correctly.***
2.2.3	Assuming that $\Delta\varphi(x)$ is linear for each ruler $$\Delta\varphi_1 = a_1 x + b_1,$$ $$\Delta\varphi_2 = a_2 x + b_2,$$ calculate the numerical values of theabove coefficients for rulers 1 and 2. (1.0 point)
2.2.4	Using those data obtained in parts 2.2.1-2.2.3, calculate the theoretical values of the intensity of light passing through the two rulersstacked together. Put down the formula that you have used for calculations. Plot the theoretical dependence in the same figure from part 2.2.1. (1.2 point)

Part 2.3 Liquid crystal cell(4.5 points)

Light transmission through LCC

Place the LCC between the polarizers as shown in Fig. 6B.

The experimental dependence under investigation is strongly nonmonotonicwith domains of quite abrupt changes. Take this into account when taking measurements.
To measure the AC voltage of the cellpower supply and the DC voltage of the photodetector, connect the appropriate leads directly to the multimeter.

2.3.1	Measure the intensity of the transmitted light as a function of the voltage across the LCC. Plot the corresponding graph. (2.0 points)
2.3.2	Calculate the phase difference between the ordinary and extraordinary waves $\Delta\varphi_0$when the power supply is disconnected with the LCC. (1.5 points)
2.3.3	In a sufficiently wide range of the voltage dropacross the LCC, the phase difference between the ordinary and extraordinary waves depends on the voltage applied by the power law $$\Delta\varphi = CU^{\beta}.$$ Using the data obtained, plot the graphwhich allows one to determine the applicability range of the above formula and calculate the exponentβ. Specify that range of applicability and evaluate the numerical value of the parameterβ. (1.0 point)

Part 2.4.Light transmission through a curved strip (3.4 points)

Secure the plastic strip on the screen as described in part 1.3.

2.4.1	Measure the intensity of light, transmitted through the optical system,as a function of the coordinate$x$of thepoint of light penetration into the strip in the range of $\pm 20\ mm$from its center. Plot the corresponding graph. (1.2 point)
2.4.2	Calculate the phase shift between the ordinary and extraordinary waves$\Delta\varphi_0$, passing through anuncurved strip. It is known that $\Delta\varphi_0$ lies in the range of$10\pi$to 12π. (1.2 points)

Near the center of the strip its shape may be approximated a circular arc of radius R. The theoretical dependence of the phase shift $\Delta\varphi$ on the distance z from the center of the strip, $z \ll R$, has the form:

$$\Delta\varphi=\Delta\varphi_0\left(1+\frac{z^2}{2n^2R^2}\right),$$

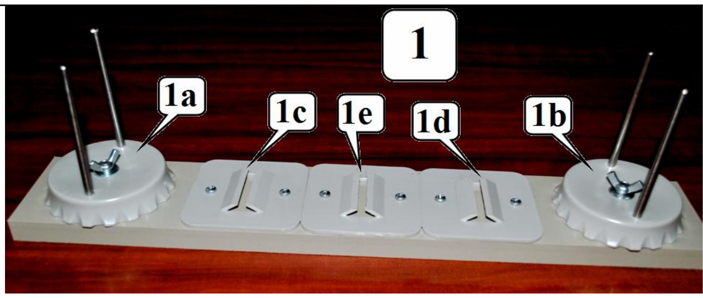
where$n = 1.4$, is the refractive index of the strip material

2.4.3	Using the data obtained in the previous parts, calculate the radius of strip curvature R near its center. The refractive index of the strip material is equal to $n = 1.4$. (1.0 points)

Appendix A. Experimental equipment

Optical bench 1with holders:

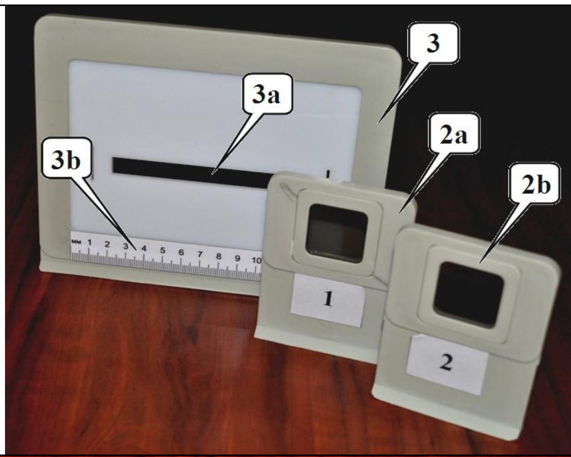
1a – support for the light source with the screw;
1b – support for the photodetector with the screw;
1c, 1d – stands for the polarizers;
1e – stand for the screen and the liquid crystal cell (LCC).



2a, 2b – polarizers on the stands. There are numbers 1 and 2 on their face sides.

The polarizers should be installed with their face sides towards the light source!

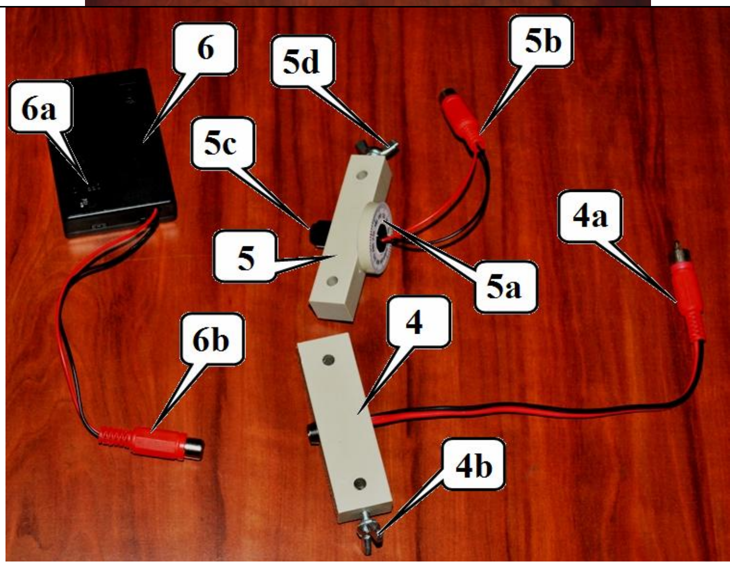
The transmission planes of the polarizers makes the angle 45°with the horizontal.
3 – the screen with the slit (3a) and the scale (3b)



Light sources:

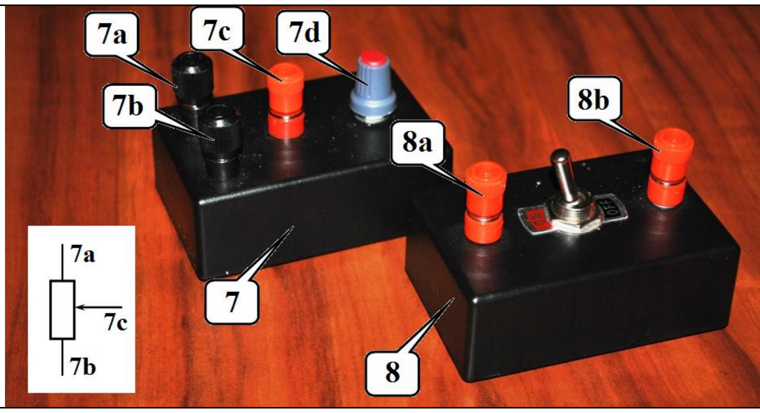
4 – light emitting diode (LED):
4a – leads for the power supply;
4b – fixing screw;
5 – laser:
5a – ring for turning the laser with the scale(scale is not used);
5b – leads for the power supply;
5c – the screw for the beam width adjustment on the front side;
5d – fixing screw;
6 – power supply for light sources:
6a – switch;
6b – leads for light source.

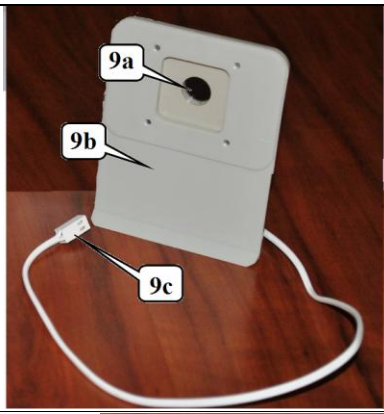
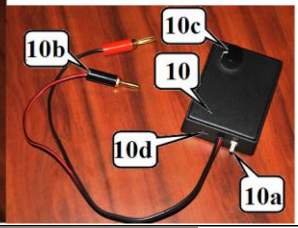
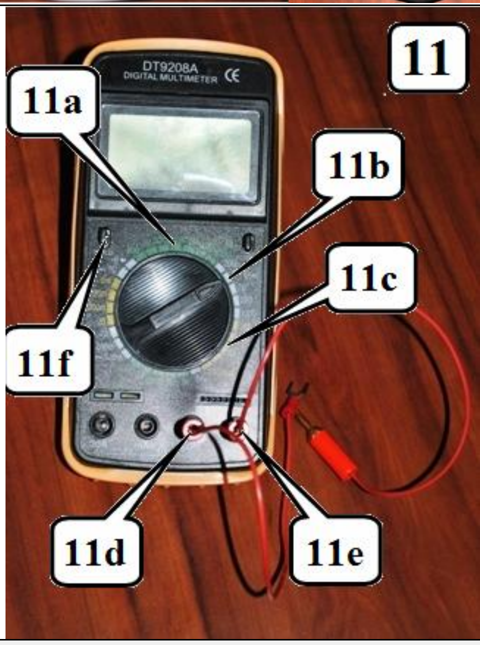
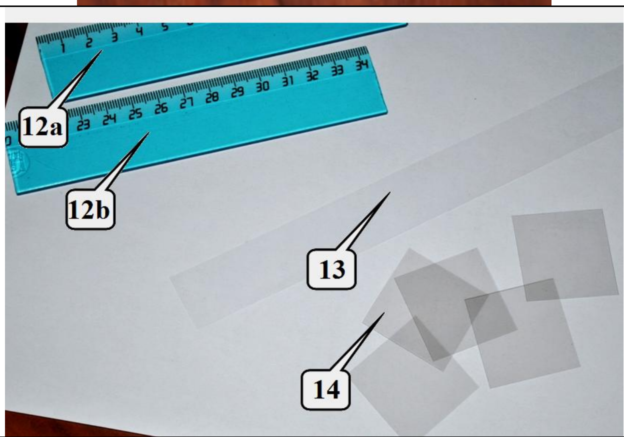
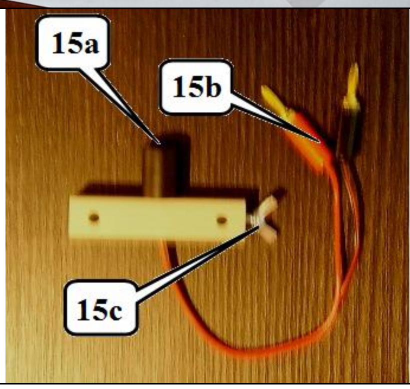
Keep the source operating only while making measurements!
Do not point the laser beam in youror anyone's eyes, it is very dangerous!



7 – variable resistor:
7a, 7b, 7c – terminals for connection to a circuit
7d – knob for changing resistance.

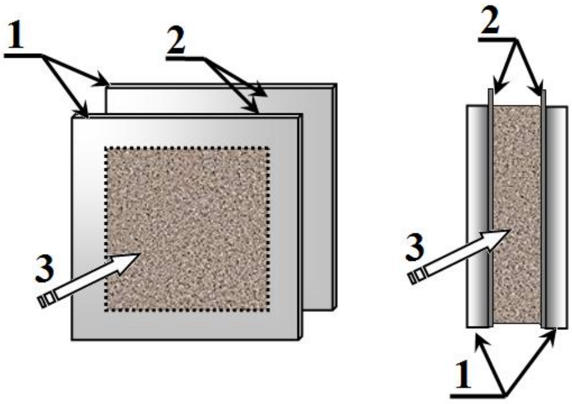
8 – switch:
8a, 8b – terminals for connection to a circuit;



9a – liquid crystal cell (LCC) 9ain a holder (9b), 9c – lead for connecting to the power supply; **10 – the power supply for LCC**: 10a – connector for the LCC; 10b – leads for measuring the output voltage; 10c – knob to adjust the output voltage; 10d – power on/off. ***Keep the source operating only while making measurements!***	 
11 – multimeter; Do not press the HOLD button 11a – register to measure resistance (200 kOhm); 11b – register to measure DC voltage (2V); 11c – register to measure AC voltage (20V); 11d, 11e – connectors for test leads; 11f –power on/off. ***If the display multimeter is in a "sleep" mode - double-press power on/off!*** ***When measuring the resistance with the multimeter, the element must be disconnected with a power supply!***	
Optical elements to be investigated 12 – plastic rulers: 12a –No. 1 (with the scalefrom 0 to 14 cm) 12b –No. 2 (with the scalefrom20 to34cm) 13 – flexible strip; 14 – set of identical filters; ***The filters and the strip are provided in a separate envelope!*** The plastic rulers and the strip exhibit birefringence, their optical axes lie in theirown planes.	
15 Photodetector (photodiode) 15a - input window; 15b - leads for measuring the output voltage; 15c –fixing screw.	
Connecting wires, clamps, paper napkin, a piece of paper.	

The liquid crystal cell is composed of two glass plates 1 whose inner surface is coated with a transparent conductive layer 2. Between the plates there is a thin (approximately 10 microns) layer of the solution 3 which is in a liquid crystal state. Leads are soldered to the plates for connecting to the AC power supply. In the absence of voltage long molecules of liquid crystal are oriented parallel to the plates. The direction of molecular orientation coincides with the optical axis of the crystal.

Try not to touch those parts of the optical elements through which the light passes! If necessary, wipe them with a paper napkin!



Appendix B. Photos of the experimental setups

Fig. 1B Setup for the observation of birefringence in the ruler

Fig. 2B Mounted ruler on the screen

Fig. 3B Fixing flexible plastic strip to the screen.

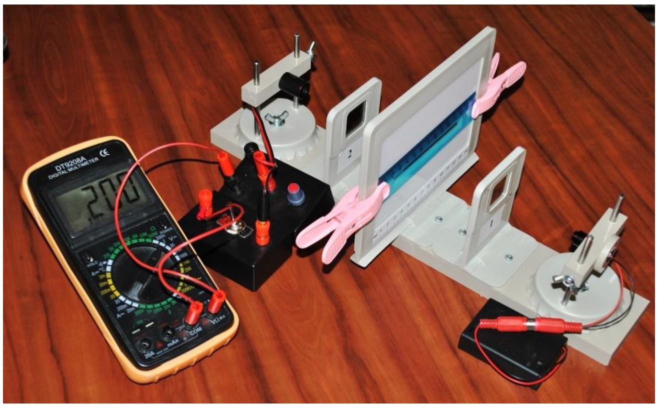


Fig. 4B Setup for measuring the transmittance of light through plastic ruler

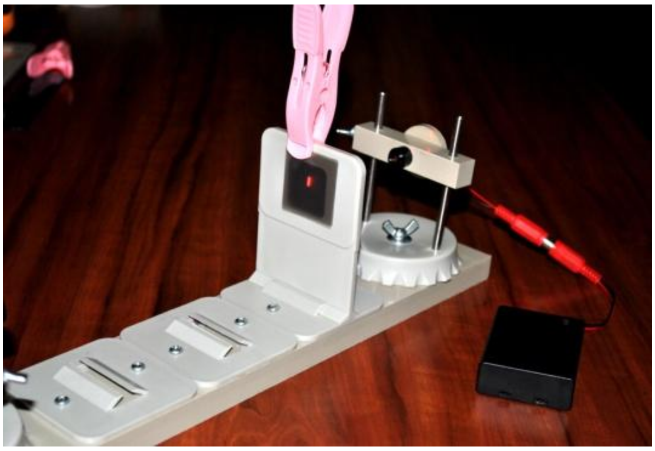


Fig. 5B Mounted filters on the polarizer

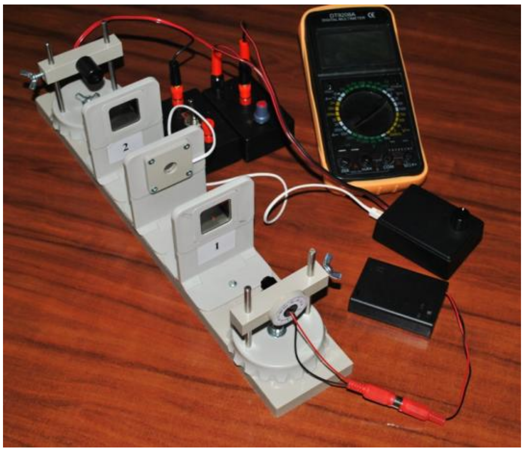


Figure 6B Setup for measuring the characteristics of the LCC