**I.1**. Equation of motion for the magnet is

$$m\ddot{z} = mg - k\dot{z} \tag{1}$$

For terminal velocity $\ddot{z} = 0$ $\boxed{0.2 \text{ mark}}$

which gives $v_T = \dot{z} = \dfrac{mg}{k}$ $\boxed{0.3 \text{ mark}}$

**I.2**. Rewriting Eq. (1)

$$\frac{dv}{dt} = g - \frac{k}{m}v(t)$$

Given that $v(t=0) = 0; z(t=0) = 0$ which yields

$$v(t) = \frac{mg}{k}(1 - e^{-kt/m}) = \frac{dz}{dt} \qquad \boxed{0.5 \text{ mark}}$$

$$\int_0^z dz = \int_0^t \frac{mg}{k}(1 - e^{-kt/m})dt$$

$$z(t) = \frac{mg}{k}\left[t + \frac{m}{k}(e^{-kt/m} - 1)\right] \qquad \boxed{0.5 \text{ mark}}$$

**I.3**. **Method - I** : Because of the relative speed $v$ between the magnet and the ring, in the field $\vec{B} = B_z\hat{k} + B_\rho\hat{\rho}$ of the magnet, the induced emf is given by

$$e_i = \int (\vec{v} \times \vec{B}).d\vec{l} \qquad \boxed{0.8 \text{ mark}}$$

$$e_i = vB_a 2\pi a \qquad \boxed{0.4 \text{ mark}}$$

where

$$B_a = \frac{\mu_0}{4\pi}\frac{3pa(z_0 - z)}{[a^2 + (z_0 - z)^2]^{5/2}} \tag{2}$$

$\boxed{0.3 \text{ mark}}$

**Method - II** :Magnetic flux ($\phi$) through the ring is

$$\phi = \int_0^a B_z 2\pi\rho d\rho \qquad \boxed{0.2 \text{ mark}}$$

$$= 2\pi\int_0^a \frac{\mu_0}{4\pi}\frac{\rho p}{(\rho^2 + (z_0 - z)^2)^{3/2}}\left[\frac{3(z_0 - z)^2}{\rho^2 + (z_0 - z)^2} - 1\right]d\rho \qquad \boxed{0.2 \text{ mark}}$$

$$\phi = \frac{\mu_0 pa^2}{2(a^2 + (z_0 - z)^2)^{3/2}} \qquad \boxed{0.3 \text{ mark}}$$

Induced emf $e_i = \dfrac{-d\phi}{dt} = -v\dfrac{d\phi}{dz}$ $\boxed{0.4 \text{ mark}}$

which gives $e_i = \dfrac{\mu_0 3pa^2 v(z_0 - z)}{2[a^2 + (z_0 - z)^2]^{5/2}}$ $\boxed{0.4 \text{ mark}}$

**I.4**. $B_z$ component will cause a radially outward force on the ring and by symmetry this yields a null force. 0.4 mark

Only $B_\rho$ will contribute to

$$\vec{df}_{em} = i(\vec{dl} \times \vec{B})$$
$$\left|\vec{f}_{em}\right| = i2\pi a B_a$$ 0.6 mark

where $B_a$ is given by Eq. (2).

**I.5**. By Newton's third law, equal and opposite force will be exerted by the ring on the magnet. Hence the magnitude of the force on the magnet by the ring is $f_{em}$. 0.5 mark

**I.6**. $e_i = L\dfrac{di}{dt} + iR$ 0.5 mark

**I.7**. Potential energy is converted to three parts:

(a) $mv^2/2$ (kinetic energy) 0.3 mark

(b) $Li^2/2$ (magnetic energy) 0.3 mark

(c) $i^2R\Delta t$ (Joule loss due to the current in time $\Delta t$). 0.4 mark

**I.8**. The magnetic field does no work in the process.

| | |
|---|---|
| Yes | |
| No | ✓ |

0.5 mark

**I.9**. Resistance of the ring

$$\Delta R = \frac{2\pi a}{\sigma w \Delta z'}$$ 0.5 mark

**I.10**. Now, the net force on the magnet, due to one ring at $z'$ is given by

$$f_{em} = (2\pi a) i B'_a$$

where

$$B'_a = \frac{\mu_0}{4\pi} \frac{3pa(z'-z)}{(a^2 + (z'-z)^2)^{5/2}}$$ 0.3 mark

and $i$ is the induced current in the ring which is given by

$$i = \frac{e_i}{\Delta R} = \frac{\sigma w e_i}{2\pi a} \Delta z'$$ 0.5 mark

Then the net force on the magnet due to the entire pipe is given by

$$F = \int_{-\infty}^{\infty} f_{em} = \int_{-\infty}^{\infty} B'^2_a (2\pi a) w\sigma dz'.\dot{z}$$ 0.2 mark

Since the pipe is very long the limits of integration can be taken as $-\infty$ and $\infty$. Substituting $B'_a$, we get

$$F = \left(\frac{\mu_0}{4\pi}\right)^2 18p^2a^3\pi w\sigma\dot{z}\int_{-\infty}^{\infty}\frac{(z'-z)^2}{((z'-z)^2+a^2)^5}dz' \qquad \boxed{0.5 \text{ mark}}$$

Let $u = (z'-z)/a$. Finally,

$$F = \left(\frac{\mu_0}{4\pi}\right)^2 \frac{18p^2\pi\sigma w\dot{z}}{a^4}\int_{-\infty}^{\infty}\frac{u^2}{(1+u^2)^5}du$$

Thus damping parameter

$$k = \left(\frac{\mu_0}{4\pi}\right)^2 \frac{18p^2\pi\sigma w}{a^4}\int_{-\infty}^{\infty}\frac{u^2}{(1+u^2)^5}du \qquad \boxed{0.5 \text{ mark}}$$

**I.11**. Given that,

$$k = f(\mu_0, p, R_0, a)$$

Dimensions of various parameters involved are

$$[\mu_0] = I^{-2}MLT^{-2} \qquad \boxed{0.2 \text{ mark}}$$
$$[p] = IL^2 \qquad \boxed{0.1 \text{ mark}}$$
$$[R_0] = I^{-2}ML^2T^{-3} \qquad \boxed{0.2 \text{ mark}}$$
$$[a] = L$$
$$[k] == MT^{-1}$$

which gives

$$k = \frac{p^2\mu_o^2}{a^4R_0} \qquad \boxed{0.5 \text{ mark}}$$