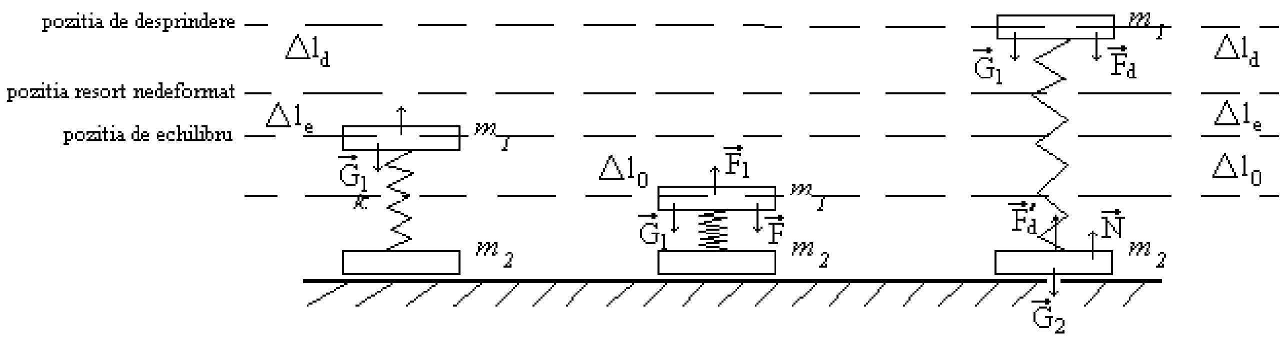
Soluție - Problema a V - a

Plăci săltărețe



<u>Forta minima</u>

a) $F_e=G_1$ → $k\Delta l_e=m_1g$ → $\Delta l_e=\dfrac{m_1g}{k}$

b) $F+G_1=F_1$ → $F=k(\Delta l_e+\Delta l_0)-m_1g$ → $F=k\Delta l_0$

c) Conditia de desprindere

$F_d+N=G_2$ → $N=G_2-F_d=m_2g-k\Delta l_d=0$ → $\Delta l_d=\dfrac{m_2g}{k}$

d) Din conservarea energiei $E_1=E_d$ unde

$$E_1^{(1)}=\frac{m_1v_{10}^2}{2}+\frac{k}{2}(\Delta l_e+\Delta l_0)^2-m_1g(\Delta l_e+\Delta l_0)$$

$$E_d^{(1)}=\frac{m_1v_d^2}{2}+\frac{k}{2}(\Delta l_d)^2+m_1g\Delta l_d$$

Cu $v_{10}=0$, rezulta

$$\frac{m_1}{2}v_d^2=(\Delta l_e+\Delta l_0+\Delta l_d)\left[\frac{k}{2}(\Delta l_e+\Delta l_0-\Delta l_d)-m_1g\right]$$

Conditia $v_d^2 \geq 0$ duce la

$$\frac{k}{2}\left(\Delta l_e + \Delta l_0 - \Delta l_d\right) - m_1 g \geq 0$$

Sau

$$\Delta l_0 \geq \frac{2 m_1 g}{k} - \Delta l_e + \Delta l_d$$

Se obtine

$$F \geq \left(m_1 + m_2\right) g$$

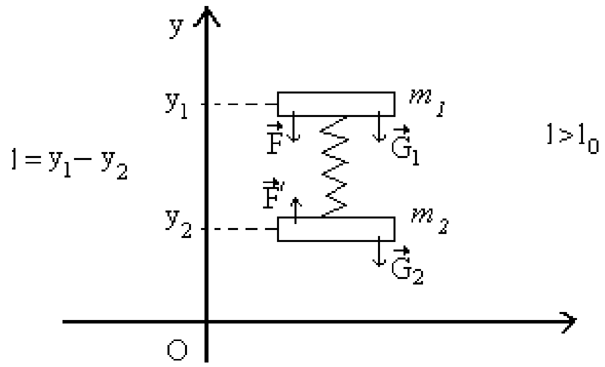
1) In cazul in care $F' = nF = n(m_1 + m_2)g$ din conservarea energiei rezulta

$$\frac{m_1}{2} v_{1d}^2 = \left(\Delta l_e + \Delta l_0 + \Delta l_d\right)\left[\frac{k}{2}\left(\Delta l_e + \Delta l_0 - \Delta l_d\right) - m_1 g\right] =$$

$$\left(\frac{m_1 g}{k} + \frac{n\left(m_1 + m_2\right)g}{k} + \frac{m_2 g}{k}\right)\left[\frac{k}{2}\left(\frac{m_1 g}{k} + \frac{n\left(m_1 + m_2\right)g}{k} - \frac{m_2 g}{k}\right) - m_1 g\right] = \frac{g^2\left(m_1 + m_2\right)^2}{2k}\left(n^2 - 1\right)$$

$$V_{1d} = \left(m_1 + m_2\right) g \sqrt{\frac{n^2 - 1}{m_1 k}}$$

2)



Din Legea a IIa:

$$m_1 a_{1y} = -k\left(l - l_0\right) - m_1 g$$
$$m_2 a_{2y} = k\left(l - l_0\right) - m_2 g$$

Adunand cele doua ecuatii:

$$m_1 a_{1y} + m_2 a_{2y} = -\left(m_1 + m_2\right) g$$

se obtine:

$$a_y^{cm} = \frac{m_1 a_{1y} + m_2 a_{2y}}{m_1 + m_2} = -g$$

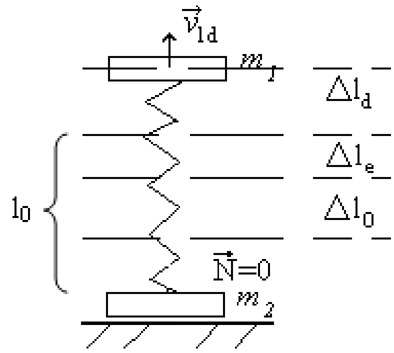
Atunci

$$v_y^{cm} = v_{0y}^{cm} - gt$$

$$y^{cm} = y_0^{cm} + v_{0y}^{cm} t - \frac{1}{2} g t^2$$

Respectiv

$$\left(v_y^{cm}\right)^2 = \left(v_{0y}^{cm}\right)^2 - 2g\left(y^{cm} - y_0^{cm}\right)$$



Cum

$$v_y^{cm} = \frac{m_1 v_{1y} + m_2 v_{2y}}{m_1 + m_2}$$

Atunci

$$v_{0y}^{cm} = \frac{m_1 v_{1d} + 0}{m_1 + m_2} = m_1 g\sqrt{\frac{n^2 - 1}{m_1 k}}$$

Din $v_y^{cm} = 0$

Rezulta

$$\Delta h_{cm} = \frac{\left(v_{0y}^{cm}\right)^2}{2g} = \frac{1}{2g} m_1^2 g^2 \frac{n^2 - 1}{m_1 k} = \frac{m_1 g}{2k}\left(n^2 - 1\right)$$

3)

$$a_{1y} - a_{2y} = \frac{-k\left(y_1 - y_2 - l_0\right) - m_1 g}{m_1} - \frac{k\left(y_1 - y_2 - l_0\right) - m_2 g}{m_2} = -k\left(\frac{1}{m_1} + \frac{1}{m_2}\right)\left(y_1 - y_2 - l_0\right)$$

Notand elongatia $u_y = l - l_0 = y_1 - y_2 - l_0$,

respectiv masa redusa $m_r = \dfrac{m_1 m_2}{m_1 + m_2}$ atunci

$$\frac{d^2 u_y}{dt^2} = \frac{d^2 y_1}{dt^2} - \frac{d^2 y_2}{dt^2} = a_{1y} - a_{2y}$$

Si deci se obtine

$$\frac{d^2 u_y}{dt^2} = -\frac{k}{m_r} u_y$$

$$u_y(t) = A\sin(\omega t + \gamma)$$

$$\frac{du_y}{dt} = \omega A\cos(\omega t + \gamma)$$

Astfel incat

$$T = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}$$

Din conditiile initiale la momentul *t=0* (desprinderea):

$$A\sin\gamma = u_y(0) = \Delta l_d$$

$$\omega A\cos\gamma = v_{1y}(0) - v_{2y}(0) = v_{1d}$$

$$A\sin\gamma = \Delta l_d$$

$$A\cos\gamma = \frac{v_{1d}}{\omega}$$

$$A = \sqrt{(\Delta l_d)^2 + \frac{v_{1d}^2}{\omega^2}}$$

$$A = \sqrt{\left(\frac{m_2 g}{k}\right)^2 + \frac{m_1 \cdot m_2}{k(m_1 + m_2)}\frac{(m_1 + m_2)^2 g^2}{m_1 k}\left(n^2 - 1\right)} = \frac{m_2 g}{k}\sqrt{1 + \left(n^2 - 1\right)\frac{m_1 + m_2}{m_2}}$$

4)

$$\tau = t_u^{cm} = \frac{v_{cm}^o}{g} = \sqrt{\frac{m_1}{k}\left(n^2 - 1\right)}$$

$$N = \left[\frac{\tau}{T}\right]_- = \left[\frac{\sqrt{\frac{m_1}{k}\left(n^2 - 1\right)}}{2\pi\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}}\right]_- = \left[\frac{1}{2\pi}\sqrt{\frac{m_1 + m_2}{m_2}\left(n^2 - 1\right)}\right]_-$$

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