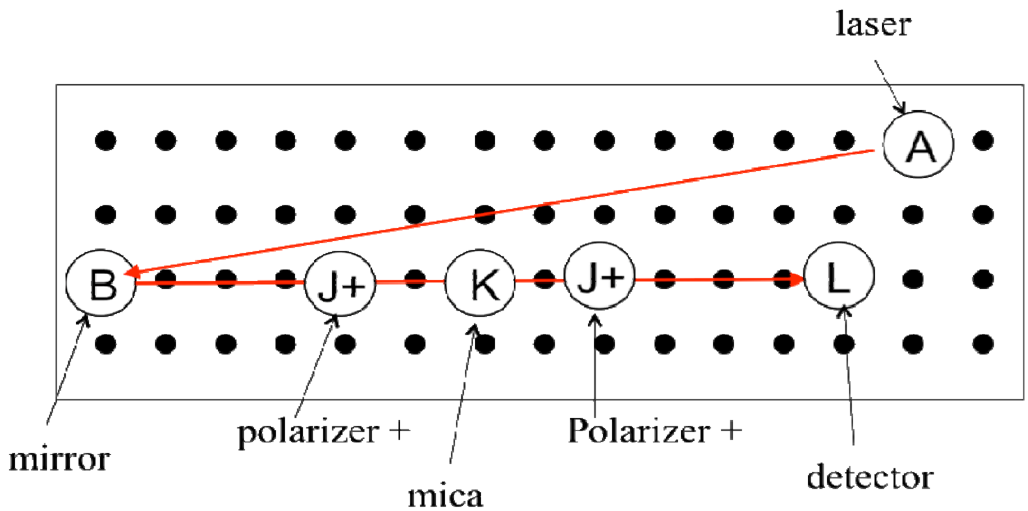
Answer Form

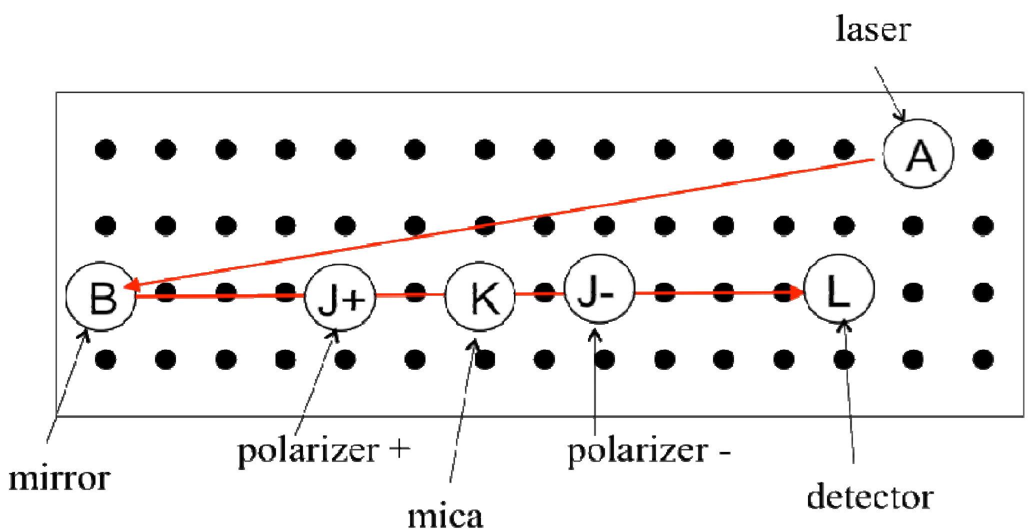
Experimental Problem No. 2

Birefringence of mica

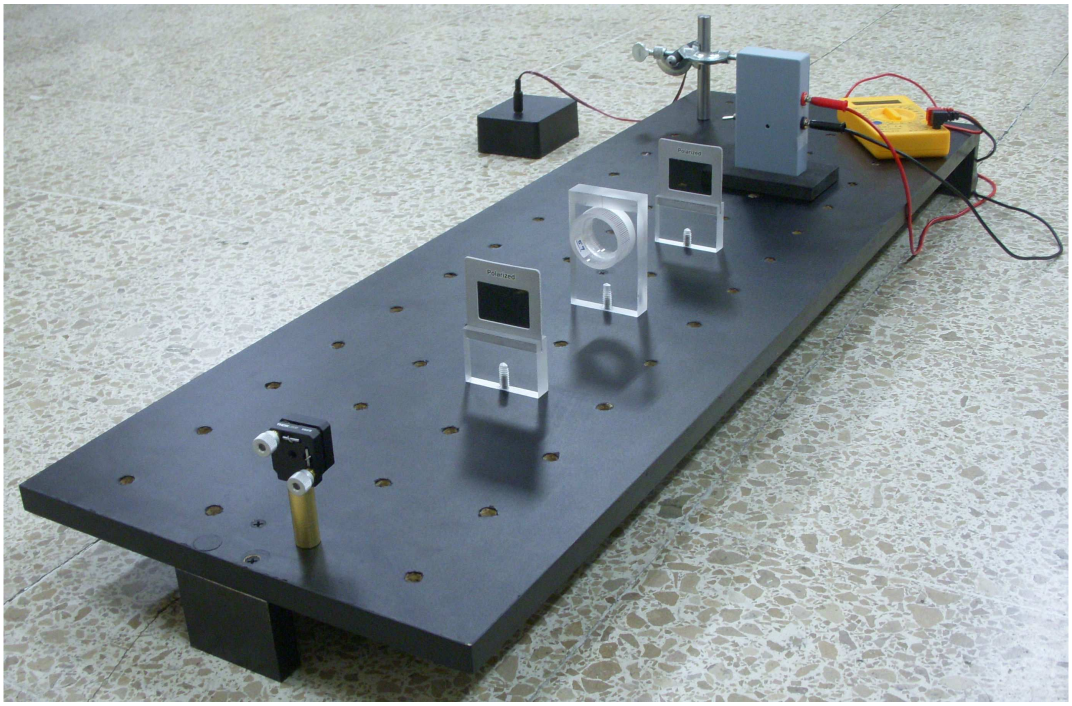
Task 2.1 a) Experimental setup for I_P. (0.5 points)



Task 2.1 b) Experimental setup for I_O. (0.5 points)



2.1		1.0



Experimental setup for measurement of mica birefringence

Task 2.2 The scale for angles.

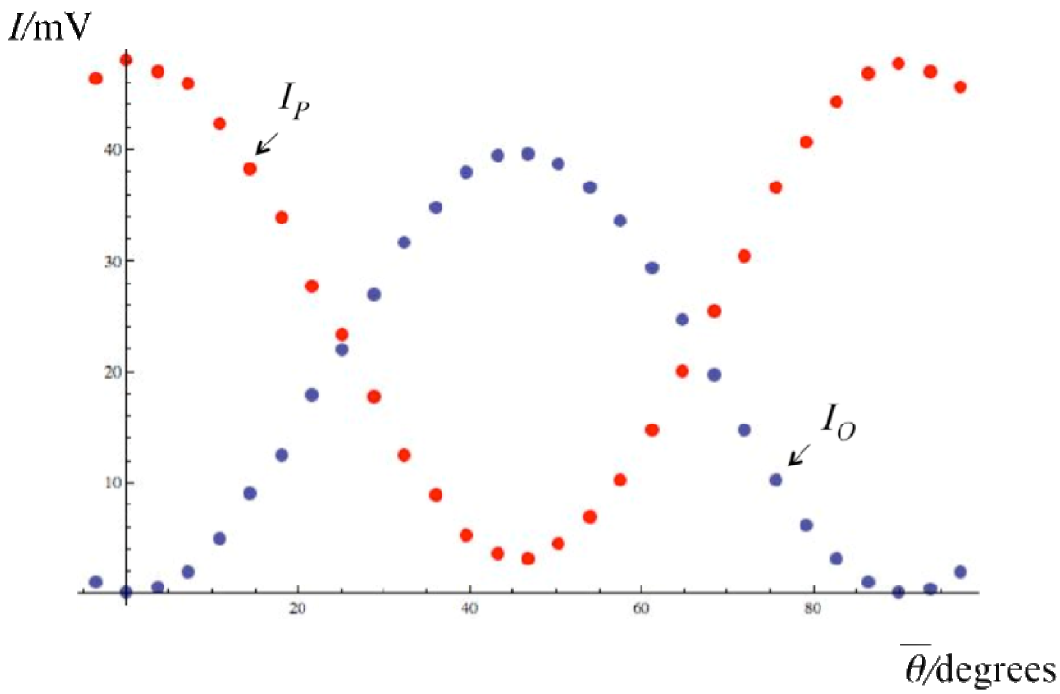
2.2	The angle between two consecutive black lines is $\theta_{\text{int}} = 3.6$ degrees because there are 100 lines.	0.25

Tasks 2.3 Measuring I_P and I_O.Use additional sheets if necessary.

TABLE I (3 points)

$\overline{\theta}$ (degrees)	$(I_P \pm 1)\times 10^{-3}$ V	$(I_O \pm 1)\times 10^{-3}$ V
-3.6	46.4	1.1
0	48.1	0.2
3.6	47.0	0.6
7.2	46.0	2.0
10.8	42.3	4.9
14.4	38.2	9.0
18.0	33.9	12.5

21.6	27.7	17.9
25.2	23.4	22.0
28.8	17.8	27.0
32.4	12.5	31.7
36.0	8.8	34.8
39.6	5.2	38.0
43.2	3.6	39.4
46.8	3.2	39.6
50.4	4.5	38.7
54.0	6.9	36.6
57.6	10.3	33.6
61.2	14.7	29.4
64.8	20.1	24.7
68.4	25.4	19.7
72.0	30.5	14.7
75.6	36.6	10.2
79.2	40.7	6.1
82.8	44.3	3.2
86.4	46.9	1.0
90.0	47.8	0.2
93.6	47.0	0.4
97.2	45.7	2.0



Parallel I_P and perpendicular I_O intensities vs angle $\overline{\theta}$.

GRAPH NOT REQUIRED!

Task 2.4 Finding an appropriate zero for θ.

2.4	*a) Graphical analysis* The value for the shift is $\delta\overline{\theta} = -1.0$ degrees. Add the graph paper with the analysis of this Task. *b) Numerical analysis* From Table I choose the first three points of $\overline{\theta}$ and $I_O(\overline{\theta})$: (intensities in millivolts) $(x_1, y_1) = (-3.6, 1.1)$ $(x_2, y_2) = (0, 0.2)$ $(x_3, y_3) = (3.6, 0.6)$ We want to fit $y = ax^2 + bx + c$. This gives three equations: $1.1 = a(3.6)^2 - b(3.6) + c$ $0.2 = c$ $0.6 = a(3.6)^2 + b(3.6) + c$ second in first $\Rightarrow$ $b = \dfrac{-0.9 + a(3.6)^2}{3.6}$ in third $\Rightarrow$ $0.6 = a\left((3.6)^2 + (3.6)^2\right) - 0.9 + 0.2$ $\Rightarrow$ $a = 0.050$ $b = -0.069$ The minimum of the parabola is at: $\overline{\theta}_{\min} = -\dfrac{b}{2a} \approx 0.7$ degrees Therefore, $\delta\overline{\theta} = -0.7$ degrees.	1.0

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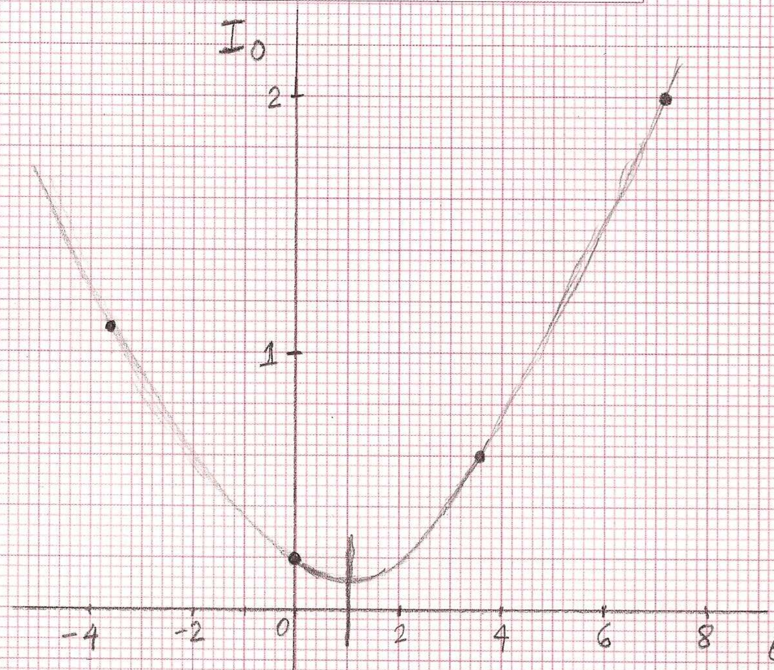


$\bar{\theta}_{min} \approx 1.0$ degrees

4 points

$\bar{\theta}$	(I_0/mV)
−3.6	1.1
0.0	0.2
3.6	0.6
7.2	2.0

Task 2.5 Choosing the appropriate variables.

2.5	Equation (2.4) for the perpendicular intensity is $\bar{I}_O(\theta) = \frac{1}{2}(1-\cos\Delta\phi)\sin^2(2\theta)$ This can be cast as a straight line $y = mx + b$, with $y = \bar{I}_O(\theta)$, $x = \sin^2(2\theta)$ and $m = \frac{1}{2}(1-\cos\Delta\phi)$ from which the phase may be obtained. **NOTE:** This is not the only way to obtain the phase difference. One may, for instance, analyze the 4 maxima of either $\bar{I}_P(\theta)$ or $\bar{I}_O(\theta)$.	0.5

Task 2.6 Statistical analysis and the phase difference.

2.6	To perform the statistical analysis, we shall then use $y = \bar{I}_O(\theta)$ and $x = \sin^2(2\theta)$.	1.0

	Since for $\theta: 0 \rightarrow \frac{\pi}{4}$, $x: 0 \rightarrow 1$, we use only 12 pairs of data points to cover this range, as given in Table II. x may be left without uncertainty since it is a setting. The uncertainty in y may be calculated as $\Delta\bar{I}_O = \sqrt{\left(\frac{\partial \bar{I}_O}{\partial I_O}\right)^2 \Delta I_O^2 + \left(\frac{\partial \bar{I}_P}{\partial I_P}\right)^2 \Delta I_P^2}$ and one gets $\Delta\bar{I}_O = \frac{\sqrt{I_O^2 + I_P^2}}{(I_O + I_P)^2} \Delta I_O \approx 0.018$, approximately the same for all values.	

TABLE II

$\bar{\theta}$ (degrees)	$x = \sin^2(2\theta)$	$y = \bar{I}_O \pm 0.018$
2.9	0.010	0.013
6.5	0.051	0.042
10.1	0.119	0.104
13.7	0.212	0.191
17.3	0.322	0.269
20.9	0.444	0.392
24.5	0.569	0.484
28.1	0.690	0.603
31.7	0.799	0.717
35.3	0.890	0.798
38.9	0.955	0.880
42.5	0.992	0.916

2.6	We now perform a least square analysis for the variables y vs x in Table II. The slope and y-intercept are: $m \pm \Delta m = 0.913 \pm 0.012$ $b \pm \Delta b = -0.010 \pm 0.008$ The formulas for this analysis are:	1.75

$$m = \frac{N\sum_{n=1}^{N} x_n y_n - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} y_{n'}}{\Delta}$$

$$b = \frac{\sum_{n=1}^{N} x_n^2 \sum_{n'=1}^{N} y_{n'} - \sum_{n=1}^{N} x_n \sum_{n'=1}^{N} x_{n'} y_{n'}}{\Delta}$$

where

$$\Delta = N\sum_{n=1}^{N} x_n^2 - \left(\sum_{n=1}^{N} x_n\right)^2$$

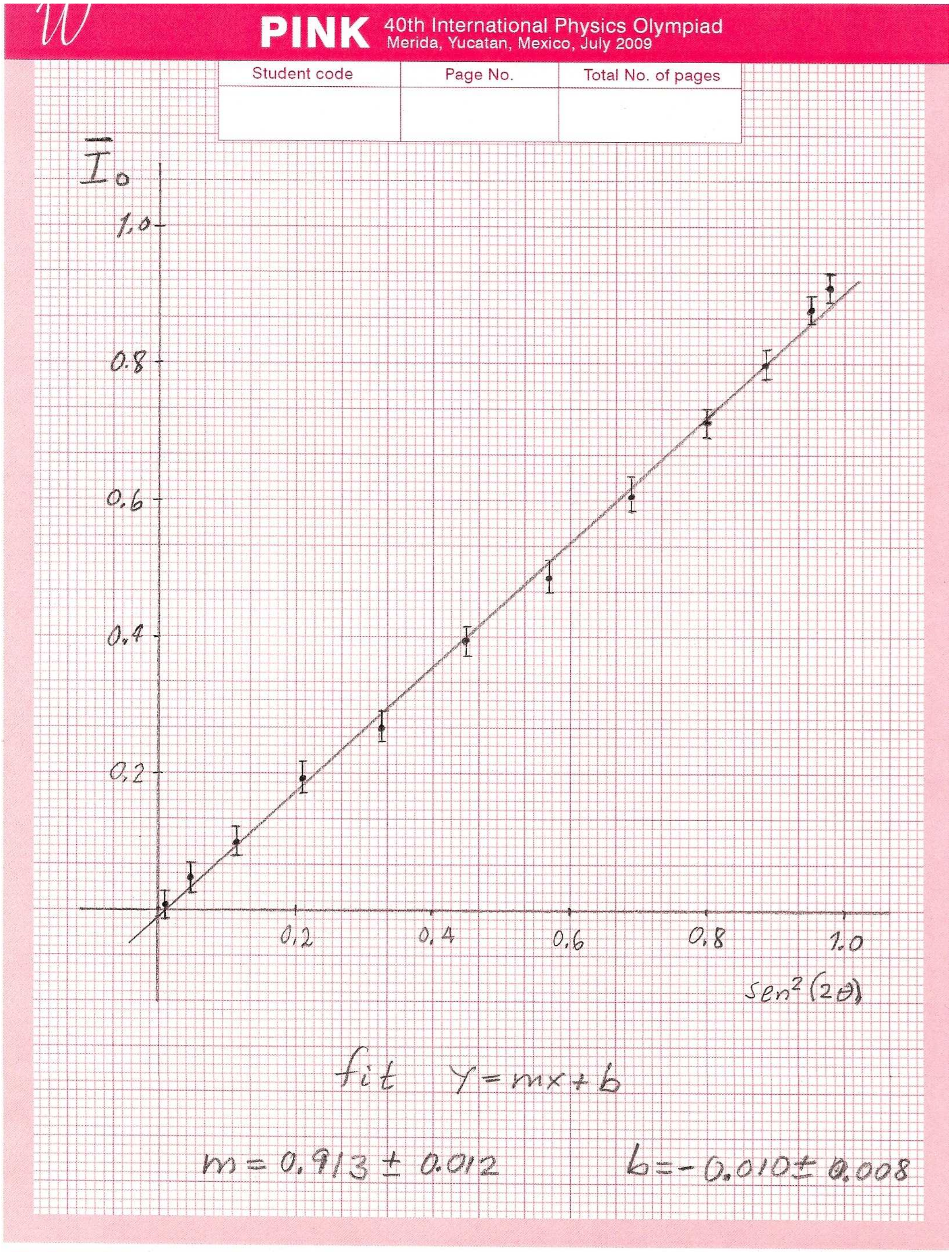
with N the number of data points.
The uncertainty is calculated as

$$(\Delta m)^2 = N\frac{\sigma^2}{\Delta} \quad , \quad (\Delta b)^2 = \frac{\sigma^2}{\Delta}\sum_{n=1}^{N} x_n^2 \qquad \text{with,}$$

$$\sigma^2 = \frac{1}{N-2}\sum_{n=1}^{N}(y_n - b - m x_n)^2$$

with $N = 12$ in this example.

Include the accompanying plot or plots.



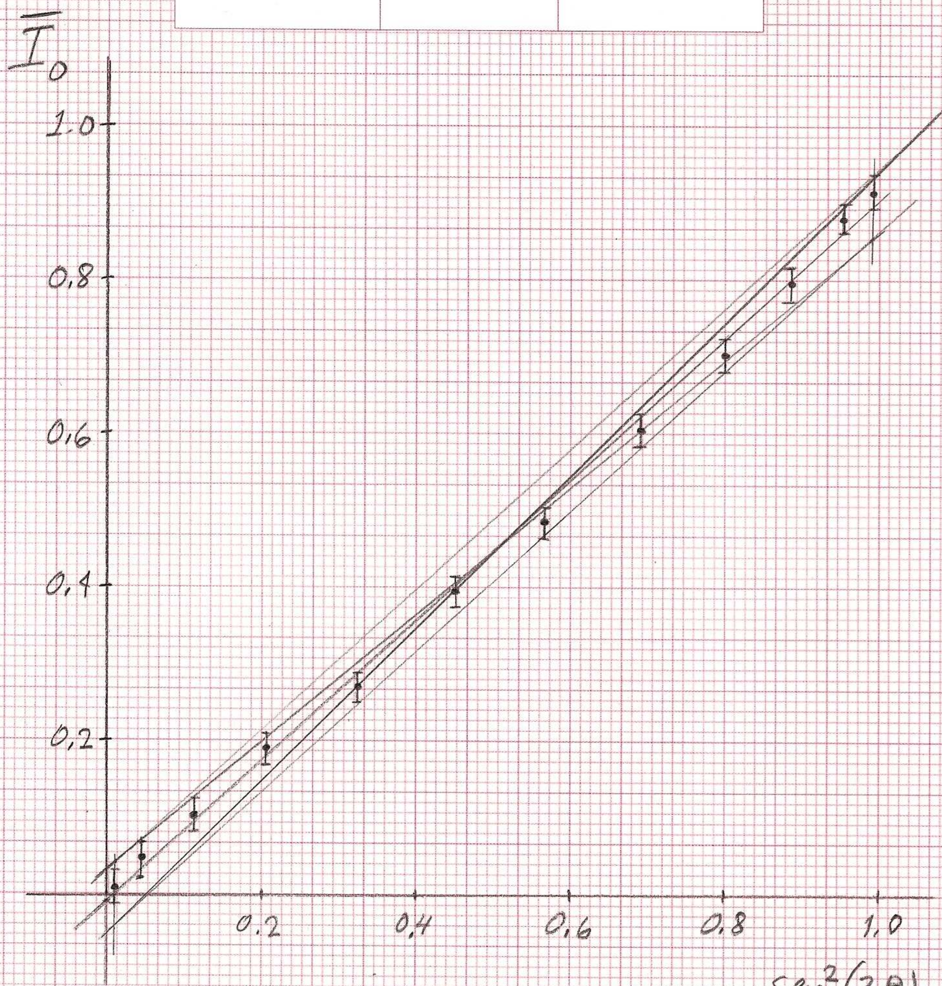
2.6	Calculate the value of the phase $\Delta\phi$ in radians in the interval $[0,\pi]$. From the slope $m=\frac{1}{2}(1-\cos\Delta\phi)$, one finds $\Delta\phi \pm \Delta(\Delta\phi) = 2.54 \pm 0.04$ Write down the formulas for the calculation of the uncertainty. We see that,	0.5

	$\Delta m = \left\lvert \frac{\partial m}{\partial \Delta\phi} \right\rvert \Delta(\Delta\phi) = \frac{1}{2}\sin(\Delta\phi)\Delta(\Delta\phi)$, therefore, $\Delta(\Delta\phi) = \frac{2\Delta m}{\sin(\Delta\phi)}$.	

Task 2.7 Calculating the birefringence $\lvert n_1 - n_2 \rvert$.

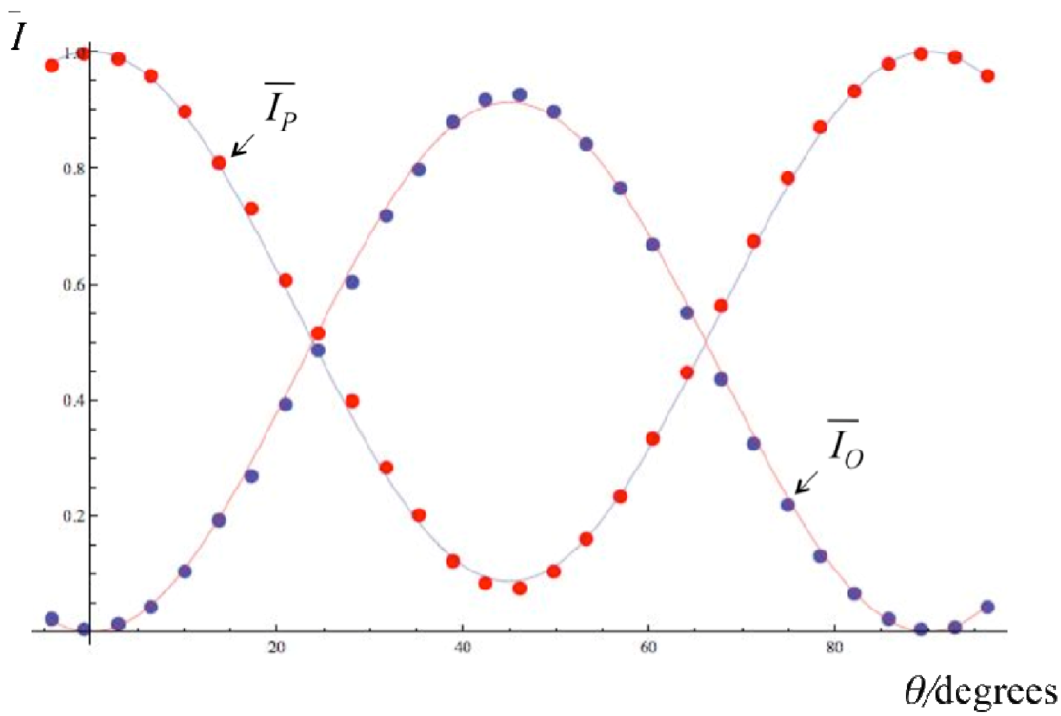
2.7	Write down the width of the slab of mica you used, $L \pm \Delta L = (100 \pm 1)\times 10^{-6}$ m Write down the wavelength you use, $\lambda \pm \Delta\lambda = (663 \pm 25)\times 10^{-9}$ m (from Problem 1) Calculate the birefringence $\lvert n_1 - n_2 \rvert \pm \Delta\lvert n_1 - n_2 \rvert = (3.94 \pm 0.16)\times 10^{-3}$ The birefringence is between 0.003 and 0.005. Nominal value 0.004 Write down the formulas you used for the calculation of the uncertainty of the birefringence. Since the width L > 82 micrometers, we use $2\pi - \Delta\phi = \frac{2\pi L}{\lambda}\lvert n_1 - n_2 \rvert$ The error is $\Delta\lvert n_1 - n_2 \rvert = \sqrt{\left(\frac{\partial \lvert n_1 - n_2 \rvert}{\partial \lambda}\right)^2 \Delta\lambda^2 + \left(\frac{\partial \lvert n_1 - n_2 \rvert}{\partial L}\right)^2 \Delta L^2 + \left(\frac{\partial \lvert n_1 - n_2 \rvert}{\partial \Delta\phi}\right)^2 \Delta(\Delta\phi)^2}$ $\Delta\lvert n_1 - n_2 \rvert = \sqrt{\left(\frac{\lvert n_1 - n_2 \rvert}{\lambda}\right)^2 \Delta\lambda^2 + \left(\frac{\lvert n_1 - n_2 \rvert}{L}\right)^2 \Delta L^2 + \left(\frac{\lambda}{2\pi L}\right)^2 \Delta(\Delta\phi)^2}$	1.0

	Since the data may appear somewhat disperse and/or the errors in the intensities may be large, a graphical analysis may be performed. In the accompanying plot, it is exemplified a simple graphical analysis: first the main slope is found, then, using the largest deviations one can find two extreme slopes. The final result is, $m = 0.91 \pm 0.08$ and $b = -0.01 \pm 0.04$ The calculation of the birefringence and its uncertainty follows as before. One now finds, $\lvert n_1 - n_2 \rvert \pm \Delta \lvert n_1 - n_2 \rvert = (3.94 \pm 0.45) \times 10^{-3}$. A larger (more realistic) error.	



Graphical analysis

$m = 0.91 \pm 0.08$ $b = -0.01 \pm 0.04$



Comparison of experimental data (normalized intensities $\bar{I}_P$ and $\bar{I}_O$) with fitting (equations (2.3) and (2.4)) using the calculated value of the phase difference $\Delta\phi$.

GRAPH NOT REQUIRED!