## Problem 1: The Earth's Horizontal Magnetic Field

This is to determine the horizontal component of the Earth's magnetic field $B_{\mathrm{H}}$ using smallamplitude oscillation of a cylindrical bar magnet. The magnet is to oscillate in the combined static fields of the Earth and that due to a square coil.


The experiment is to be done in three sections. Section I is a derivation of formulae to be used in Section III.

## Apparatus

Each student is provided with apparatus as shown in Figure 2:

1. a square coil of resistance $5.2 \pm 0.2 \Omega$ and 130 turns
2. a small cylindrical magnet of mass $15.0 \pm 0.2 \mathrm{~g}$ with nylon strings.
3. a voltmeter (for measuring the potential difference across the coil only)
4. a power supply (placed under the table to avoid the interference of its magnetic field)
5. a wooden stand
6. a stop watch
7. a protractor
8. color clay
9. a ruler
10. white label (you can write on it)
11. graph papers
12. an electrical cord


DO NOT change
these connections.

Figure 2

## Warning

Use the multi-meter to measure only the voltage difference of the coils. Using the multi-meter in other modes can destroy the power supply!!!

## Section I



Figure 3

It is given here that the magnetic flux density $B_{\mathrm{P}}$ at a perpendicular distance $\ell$ from the middle of a straight current element $i a$ is

$$
\begin{equation*}
B_{\mathrm{p}}=\frac{\mu_{0} i}{2 \pi \ell} \frac{(a / 2)}{\sqrt{\ell^{2}+\left(\frac{a}{2}\right)^{2}}} \tag{i}
\end{equation*}
$$

where $\mu_{0}=4 \pi \times 10^{-7}$ henry per metre, the permeability of free space.

Use this expression to show that the expression for the magnitude of the magnetic flux density from the square coil at point $\mathbf{P}$ in Figure 1 is given by

$$
\begin{equation*}
B_{p x}=\left(\frac{\mu_{0} a^{2} i N}{2 \pi}\right)\left[\frac{1}{\left(x^{2}+\left(\frac{a}{2}\right)^{2}\right) \sqrt{x^{2}+2\left(\frac{a}{2}\right)^{2}}}\right] \tag{ii}
\end{equation*}
$$

It is also given here that the period of a small-amplitude oscillation of the magnet in the net magnetic field $B$ is

$$
\begin{equation*}
T=2 \pi \sqrt{\frac{I}{m B}} \tag{iii}
\end{equation*}
$$

where $m$ is the magnetic moment of magnet with mass $M$, and $I$ is its moment of inertia about the axis through its centre of mass

$$
\begin{equation*}
I=M\left(\frac{L^{2}}{12}+\frac{r^{2}}{4}\right) \tag{iv}
\end{equation*}
$$

## Section II

For the experiments in Section III you have to align the magnet in the position as shown in Figure 1. If the length of the string is too small, the torsion of the string cannot be neglected in the oscillation of the magnet. Perform appropriate measurements (say, oscillation of magnet in Earth's magnetic field alone) to justify that we can ignore the torsion of the string. You are not required to plot a graph.

## Section III

For the following experiments (in a, b, and c), you have to align the magnet in the position as shown in Figure 1. Measure and write down the value of the distance between the centre of the magnet and the top surface of the platform.
a) Coil's magnetic field and Earth's horizontal magnetic field in the same direction [5 points]

## Warning <br> Please connect the coil to the power supply and leave it on for at least 5 minutes.

Measure periods of oscillation for different values of the combined field strength when the coil's magnetic field and Earth's magnetic field are in the same direction. Draw a straight line graph and compute the values of $B_{\mathrm{H}}$ and the magnetic moment $m$ from this graph and estimate their errors.
b) Earth's magnetic field only

Use the value of $m$ from (a) and the period of oscillation of the magnet bar in the absence of the Coil's magnetic field from Section II to calculate again the value for $B_{H}$ and estimate its error.
c) Coil's magnetic field and Earth's horizontal magnetic field in opposite directions [2 points] By reversing the connection at the power supply, find the equilibrium position $x_{0}$ along the Xdirection between Earth's magnetic field and the opposing magnetic field from the coil. Use the value of $x_{0}$ to calculate again the value for $B_{\mathrm{H}}$ and estimate its error.

## Problem 2: Oscillation of Water-Filled Vessel

The student is required to perform non-destructive measurements in order to determine the thickness $t$ of an aluminium vessel whose cavity is completely filled with water. The aluminium vessel is composed of a cylinder and two end plates. The cylinder is of length $L$ and outer radius $R$. The total length of the vessel is $h$. The thickness of both end plates is 0.60 cm (see Figure 1). You can neglect the error of this thickness. In this problem, please use gramme and centimetre as units for mass and length, respectively.


Figure 1


Figure 2

Figure 2 shows the so-called bifilar suspension of mass $M$. The two strings are each of equal length $\ell$. The period $T$ of a small-amplitude oscillation of $M$ is

$$
\begin{equation*}
T=2 \pi \sqrt{\frac{\ell}{g} \cdot \frac{I}{M a^{2}}} \tag{i}
\end{equation*}
$$

where $I$ is the effective moment of inertia about the vertical axis through the centre of mass of $M$ and $g$ is the acceleration due to gravity at Bangkok $\left(g=978 \mathrm{~cm} \mathrm{~s}^{-2}\right)$.

This experiment consists of two parts. Section I concerns a derivation of formulae and Section II concerns the actual experimentation.

## Apparatus

Each student is provided with:

1. a water-filled vessel
2. a stand
3. a stop watch
4. a ruler
5. a nylon string
6. a protractor
7. masking tapes
8. a knife (not shown in the figure below)


## Section I

[2.0 points]
The student is to derive expressions in terms of $R, L, t$ and the density $\rho$ of aluminium of the following quantities, [see Figure 1]
i) mass $\left(m_{1}\right)$ of the cylindrical body of the vessel,
ii) mass $\left(m_{2}\right)$ of each end plate,
iii) mass $\left(m_{3}\right)$ of water in the whole cavity,
iv) the total mass ( $M$ ) of the water-filled vessel, and
v) the effective moment of inertia, $I_{\mathrm{y}}$, about the Y-axis, of this water-filled vessel (see Figure 1), assuming that the water is ideal fluid.

Then perform measurements of $R, h, L$. By substituting the values, derive expressions in terms of $t$ for the quantities i)-v) above. The aluminium density $\rho=2.70 \mathrm{~g} / \mathrm{cm}^{3}$ and the water density is 1.00 $\mathrm{g} / \mathrm{cm}^{3}$.

## Hint:



Thin rod of length $L$


Thin cylinder of inner radius $R_{1}$ and outer radius $R_{2}$

Figure 3

## Section II



Figure 4


Figure 5
a) Angular oscillation about the axis of symmetry
[4.0 points]
For one fixed value of $\ell$, perform precise measurements of the period $T_{\mathrm{y}}$ for a small-amplitude oscillation as in Figure 4. Then compute the value of the thickness $(t)$ of the cylindrical wall. Estimate the experimental error $\Delta t$ for the thickness.

Compute also the values of $m_{1}, m_{2}, m_{3}$, and $M$ using this value of $t$.
b) Angular oscillation about the central axis perpendicular to the length
[2.8 points]
Change the bifilar suspension of the vessel to that of Figure 5 and make similar measurements as in (a).

Then use the value of the period of oscillation just found together with the values of $t, m_{1}, m_{3}, M$ found in (a) to compute the value of the effective moment of inertia $I_{\mathrm{x}}^{\mathrm{Exp}}$ of the vessel about the X-axis (see Figure 2 and Figure 5).

Compute also the theoretical estimate of the value of $I_{\mathrm{x}}^{\text {Theo }}$ based on the value of $t$ found in (a) assuming that the whole of the computed mass of water found in (a) is now constrained to take part in the oscillatory motion of the vessel.

## c) Comparing experimental and theoretical values of the moment of inertia

What is the difference $\left(\Delta I_{\mathrm{x}}\right)$ between the values of $I_{\mathrm{x}}^{\text {Theo }}$ and $I_{\mathrm{x}}^{\mathrm{Exp}}$ ?
Do you consider this difference statistically significant?
Estimate the percentage of the mass of water that takes part in the oscillatory motion in (b), assuming this water to be circular discs adhering to the end plates.

## Hint:

$$
I_{\mathrm{x}}^{\text {Theo }}=m_{1}\left[\frac{L^{2}}{12}+\frac{R^{2}+(R-t)^{2}}{4}\right]+2 m_{2}\left[\frac{(0.6 \mathrm{~cm})^{2}}{12}+\frac{R^{2}}{4}+\left(\frac{L}{2}+\frac{0.6 \mathrm{~cm}}{2}\right)^{2}\right]+m_{3}\left[\frac{L^{2}}{12}+\frac{(R-t)^{2}}{4}\right]
$$

