**SOLUTION EXPERIMENT I**

PART A

1. **[Total 0.5 pts]**
   The experimental method chosen for the calibration of the arbitrary scale is a simple pendulum method [0.3 pts]

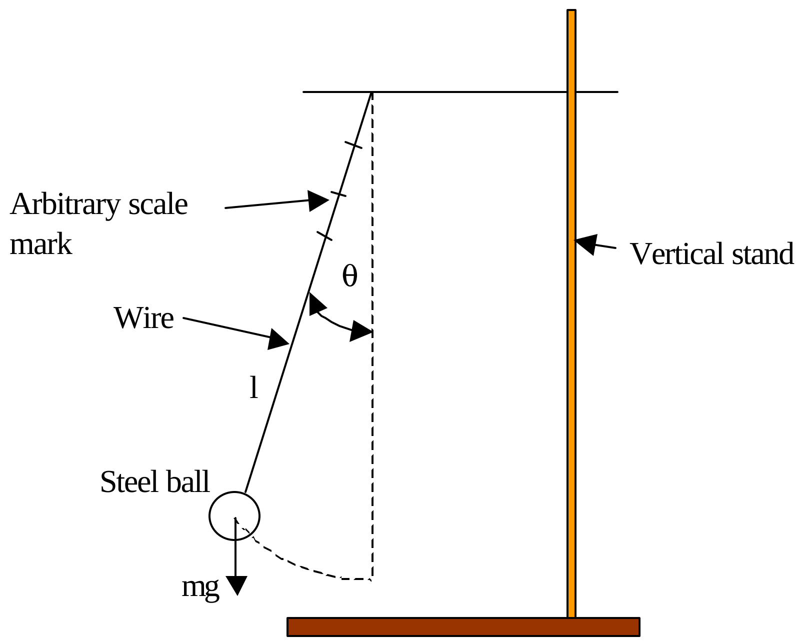



Figure 1. Sketch of the experimental set up [0.2 pts]

2. **[Total 1.5 pts]**
   The expression relating the measurable quantities: [0.5 pts]

$$T_{osc} = 2\pi\sqrt{\frac{l}{g}}\ ;\ T_{osc}^{\ 2} = 4\pi^2\frac{l}{g}$$

Approximations :

$\sin\theta \approx \theta$ [0.5 pts]

mathematical pendulum (mass of the wire << mass of the steel ball,
the radius of the steel ball << length of the wire [0.5 pts]
flexibility of the wire, air friction, etc [0.1 pts, only when one of the two major points above is not given]

3. **[Total 1.0 pts]** Data sample from simple pendulum experiment
   # of cycle ≥ 20 [0.2 pts.] , difference in T ≥ 0.01 s [0.4 pts], # of data ≥ 4 [0.4 pts]

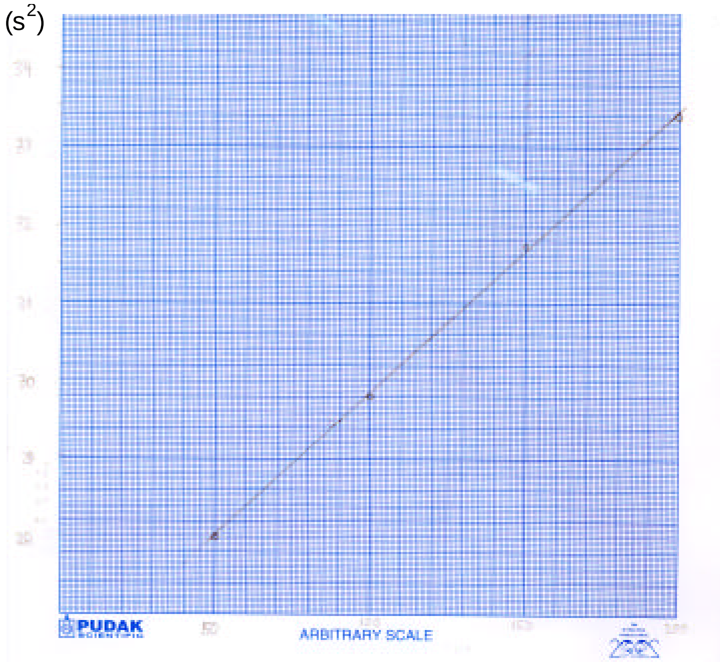
| No. | t(s) for 50 cycles | Period, T (s) | Scale marked on the wire (arbitrary scale) |
|---|---|---|---|
| 1 | 91.47 | 1.83 | 200 |
| 2 | 89.09 | 1.78 | 150 |
| 3 | 86.45 | 1.73 | 100 |
| 4 | 83.8 | 1.68 | 50 |

4. **[Total 0.5 pts]**

| No. | Period, T (s) | Scale marked on the wire (arbitrary scale) | $T^2 (s^2)$ |
|---|---|---|---|
| 1 | 1.83 | 200 | 3.35 |
| 2 | 1.78 | 150 | 3.17 |
| 3 | 1.73 | 100 | 2.99 |
| 4 | 1.68 | 50 | 2.81 |

The plot of $T^2$ vs scale marked on the wire:

$T^2$ $(s^2)$




Scale marked on the wire (arbitrary scale)

5. Determination of the smallest unit of the arbitrary scale in term of mm **[Total 1.5 pts]**

$$T_{osc_1}^2 = \frac{4p^2}{g} L_1 \ , \qquad T_{osc_2}^2 = \frac{4p^2}{g} L_2$$

$$\left(T_{osc_1}^2 - T_{osc_2}^2\right) = \frac{4p^2}{g} L_1 - L_2 = \frac{4p^2}{g} \Delta L$$

$$\Delta L = \frac{g}{4\pi^2}\left(T_{osc_1}^2 - T_{osc_2}^2\right)$$ or other equivalent expression [0.5 pts]

| No. | | Calculated $\Delta$L (m) | $\Delta$L in arbitrary scale | Values of smallest unit of arbitrary scale (mm) |
|---|---|---|---|---|
| 1. | $T_1^2$-$T_2^2$ = 0.171893 $s^2$ | 0.042626 | 50 | 0.85 |
| 2. | $T_1^2$-$T_3^2$ =0.357263 $s^2$ | 0.088595 | 100 | 0.89 |
| 3. | $T_1^2$-$T_4^2$ =0.537728 $s^2$ | 0.133347 | 150 | 0.89 |
| 4. | $T_2^2$-$T_3^2$ =0.18537 $s^2$ | 0.045968 | 50 | 0.92 |
| 5. | $T_2^2$-$T_4^2$ =0.365835 $s^2$ | 0.09072 | 100 | 0.91 |
| 6. | $T_3^2$-$T_4^2$ =0.180465 $s^2$ | 0.044752 | 50 | 0.90 |

The average value of smallest unit of arbitrary scale, $\bar{l}$ = 0.89 mm [0.5 pts]

The estimated error induced by the measurement: [0.5 pts]

| No. | Values of smallest unit of arbitrary scale (mm) | $(l - \bar{l})$ | $(l - \bar{l})^2$ |
|---|---|---|---|
| 1. | 0.85 | -0.04 | 0.0016 |
| 2. | 0.89 | 0 | 0 |
| 3. | 0.89 | 0 | 0 |
| 4. | 0.92 | 0.03 | 0.0009 |
| 5. | 0.91 | 0.02 | 0.0004 |
| 6. | 0.90 | 0.01 | 0.0001 |

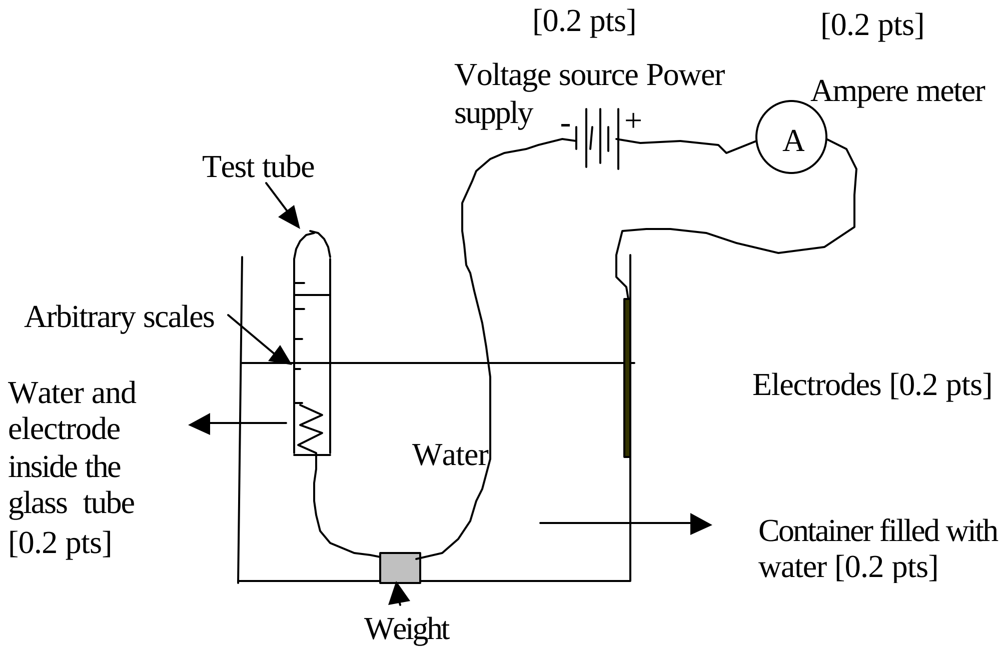
And the standard deviation is:

$$\Delta l = \sqrt{\frac{\sum_{i=1}^{6}(l-\bar{l})^2}{N-1}} = \sqrt{\frac{0.003}{5}} = 0.02 \text{ mm}$$

other legitimate methods may be used

PART B

1. The experimental set up:**[Total 1.0 pts]**




2. Derivation of equation relating the quantities time $t$, current $I$, and water level difference $\Delta h$: :**[Total 1.5 pts]**

$$I = \frac{\Delta Q}{\Delta t}$$

From the reaction: $2\,H^+ + 2\,e \longrightarrow H_2$, the number of molecules produced in the process ($\Delta$N) requires the transfer of electric change is $\Delta Q = 2e\,\Delta N$ : [0.2 pts]

$$I = \frac{\Delta N\, 2e}{\Delta t}$$ [0.5 pts]

$$P\,\Delta V = \Delta N\, k_B\, T$$ [0.5 pts]

$$= \frac{I\,\Delta t}{2e} k_B\, T$$

$$P\,\Delta h(\pi r^2) = \frac{I\,\Delta t}{2}\frac{k_B}{e} T$$ [0.2 pts]

$$I\,\Delta t = \frac{e}{k_B}\frac{2P(\pi r^2)}{T}\Delta h$$ [0.1 pts]

**3.** The experimental data: **[ Total 1.0 pts]**

| No. | $\Delta$h (arbitrary scale) | I (mA) | $\Delta$t (s) |
|---|---|---|---|
| 1 | 12 | 4.00 | 1560.41 |
| 2 | 16 | 4.00 | 2280.61 |
| 3 | 20 | 4.00 | 2940.00 |
| 4 | 24 | 4.00 | 3600.13 |

- The circumference $\phi$, of the test tube = 46 arbitrary scale [0.3 pts]
- The chosen values for $\Delta h$ ($\geq$ 4 scale unit) for acceptable error due to uncertainty of the water level reading and for $I$ ($\leq$ 4 mA) for acceptable disturbance [0.3 pts]
- \# of data $\geq$ 4 [0.4 pts]

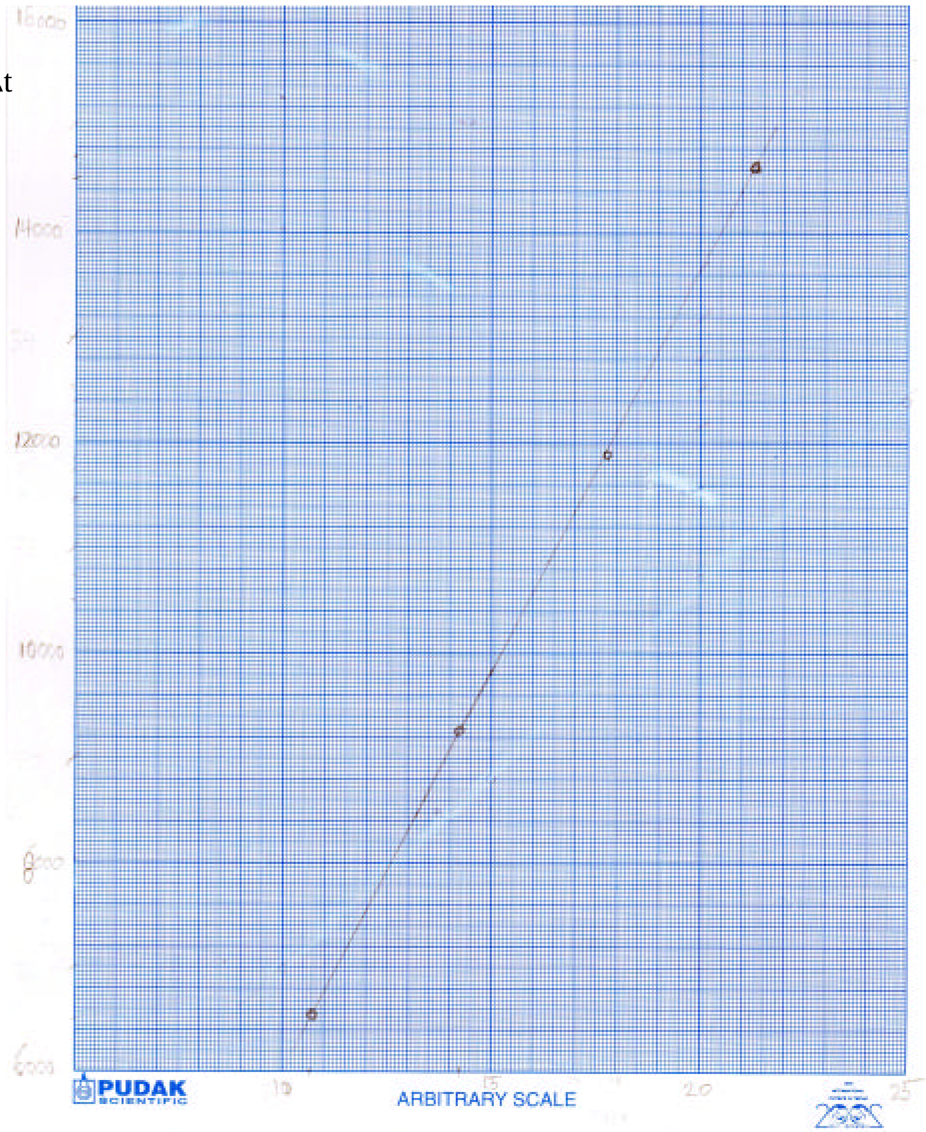
The surrounding condition ($T$,$P$) in which the experimental data given above taken:
$T$ = 300 K
$P$ = 1.00 $10^5$ Pa

4. Determination the value of $e/k_B$ **[Total 1.5 pts]**

| No. | $\Delta$h (arbitrary scale) | $\Delta$h (mm) | I (mA) | $\Delta$t (s) | I $\Delta$t ( C ) |
|---|---|---|---|---|---|
| 1 | 12 | 10.68 | 4.00 | 1560.41 | 6241.64 |
| 2 | 16 | 14.24 | 4.00 | 2280.61 | 9120.48 |
| 3 | 20 | 17.80 | 4.00 | 2940.00 | 11760.00 |
| 4 | 24 | 21.36 | 4.00 | 3600.13 | 14400.52 |

Plot of $I\Delta t$ vs $\Delta h$ from the data listed above

$I\,\Delta t$




$\Delta h$

The slope obtained from the plot is 763.94;

$$\frac{e}{k_B} = \frac{763.94 \times 300 \times \boldsymbol{p}}{2 \times 10^5 \times (23 \times 0.89 \times 10^{-3} \times 0.82)^2} = 1.28 \times 10^4 \text{ Coulomb K/J}$$

[1.0 pts]

Alternatively [the same credit points]

| No. | $\Delta h$ (mm) | $I\,\Delta t$ ( C ) | Slope | $e/k_b$ |
|---|---|---|---|---|
| 1 | 10.68 | 6241.64 | 584.4232 | 9774.74 |
| 2 | 14.24 | 9120.48 | 640.4831 | 10712.37 |
| 3 | 17.80 | 11760.00 | 660.6742 | 11050.07 |
| 4 | 21.36 | 14400.52 | 674.1816 | 11275.99 |

Average of $e/k_b = 1.07 \times 10^4$ Coulomb K/J
[1.0 pts]

| No. | $e/k_b$ | difference | Square difference |
|---|---|---|---|
| 1 | 9774.74 | -928.55 | 862205.5 |
| 2 | 10712.37 | 9.077117 | 82.39405 |
| 3 | 11050.07 | 346.7808 | 120256.9 |
| 4 | 11275.99 | 572.6996 | 327984.9 |

Estimated error [0.5 pts]

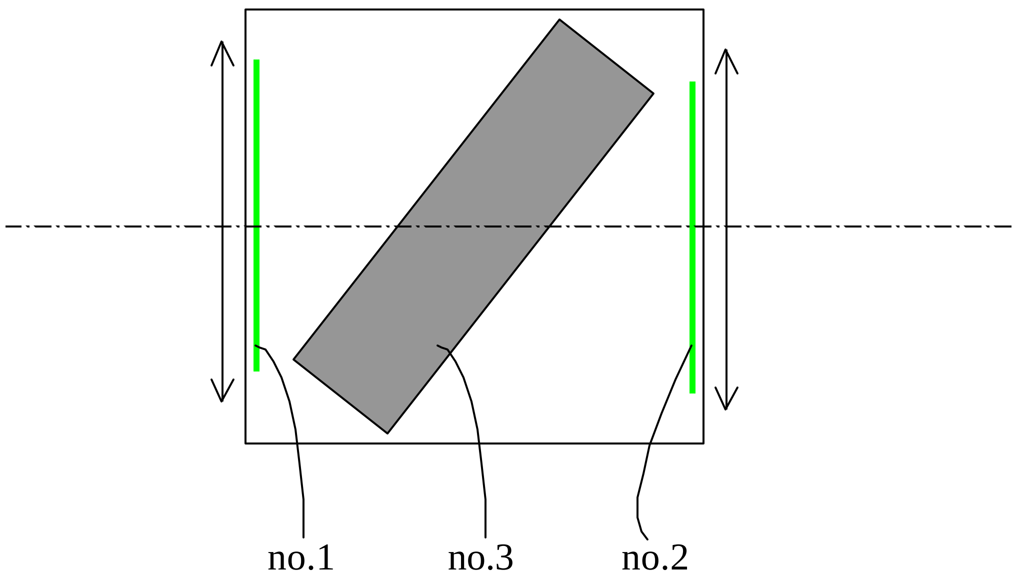
The standard deviation obtained is $0.66 \times 10^3$ Coulomb K/J,
Other legitimate measures of estimated error may be also used

SOLUTION OF EXPERIMENT PROBLEM 2

1. The optical components are **[total 1.5 pts]**:

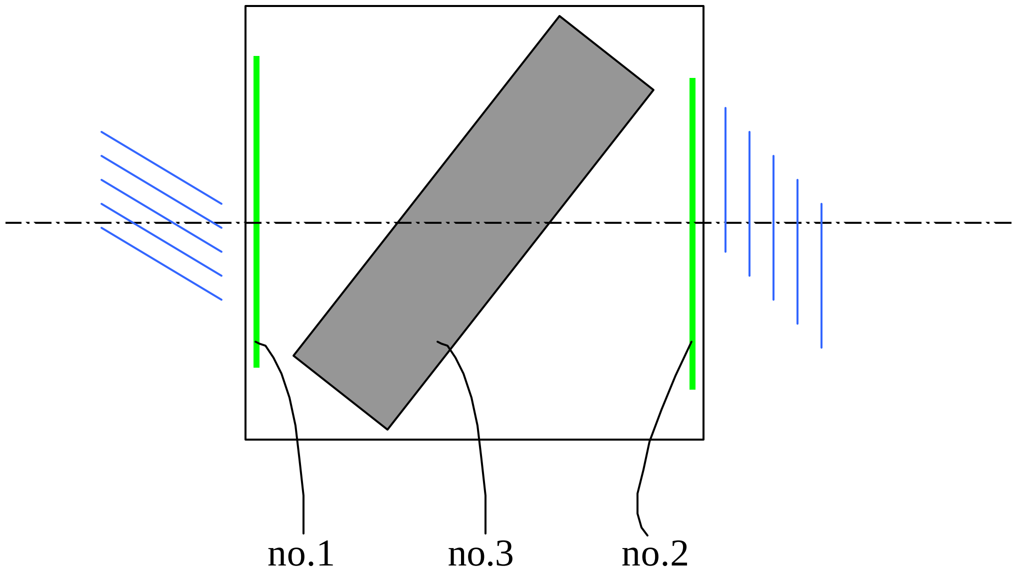
| | | |
|---|---|---|
| no.1 | Diffraction grating | [***0.5 pts***] |
| no.2 | Diffraction grating | [***0.5 pts***] |
| no.3 | Plan-parallel plate | [***0.5 pts***] |

2. Cross section of the box **[total 1.5 pts]**:




[***0.5 pts***] [***0.5 pts***] [***0.5 pts***]

3. Additional information **[total 1.0 pts]**:



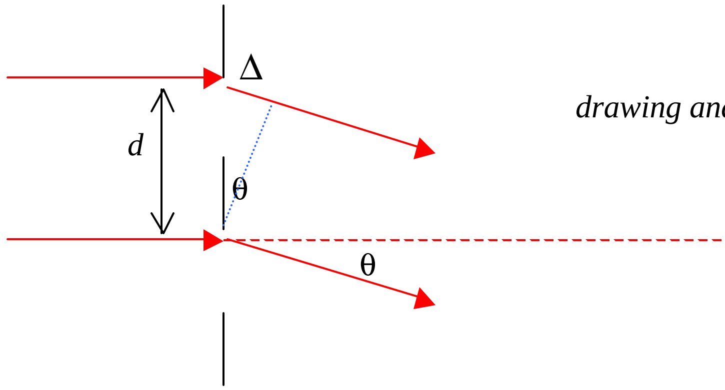

Distance of the grating (no.1) to the left wall is practically zero [***0.2 pts***]

Distance of the grating (no.2) to the right wall is practically zero [***0.2 pts***]

Lines of grating no.1 is at right angle to the slit [***0.3 pts***]

Lines of grating no. 2 is parallel to the slit [***0.3 pts***]

4. Diffraction grating **[total 2.0 pts]**:




*drawing and labels should be complete*
[***0.6 pts***]

Path length difference:

$$\Delta = d \sin\theta \ , \qquad d = \text{spacing of the grating}$$

Diffraction order:

$$\Delta = m\lambda \ , \qquad m = \text{order number}$$

Hence, for the first order ($m$ = 1):

$$\sin\theta = \lambda / d$$ [***0.4 pts***]

<u>Observation data:</u>

| $\tan\theta$ | $\theta$ | $\sin\theta$ |
|---|---|---|
| 0.34 | $18.78^0$ | 0.3219 |
| 0.32 | $17.74^0$ | 0.3048 |
| 0.32 | $17.74^0$ | 0.3048 |

*number of data* $\geq 3$
[***0.5 pts***]

| Name of component no.1 | Specification |
|---|---|
| Diffraction grating | Spacing = 2.16 μm<br>Lines at right angle to the slit |

**[*0.4 pts*]**
**[*0.1 pts*]**

Note: true value of grating spacing is 2.0 μm, deviation of the result ≤ 10%

5. Diffraction grating **[total 2.0 pts]**:

For the derivation of the formula, see nr.4 above.

[***1.0 pts***]

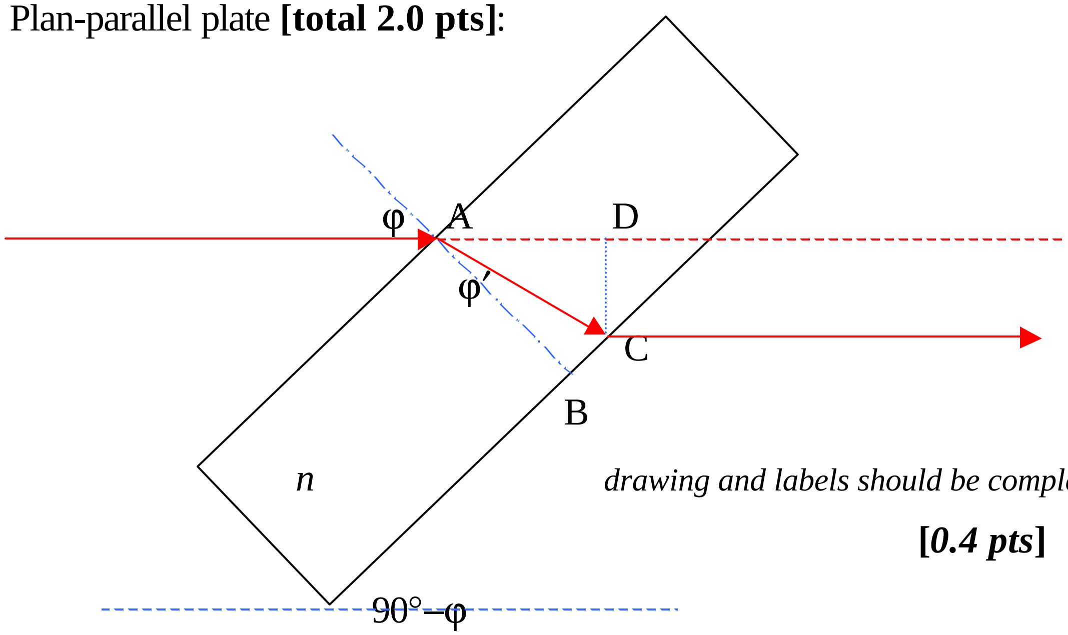
<u>Observation data:</u>

| $\tan\theta$ | $\theta$ | $\sin\theta$ | |
|---|---|---|---|
| 1.04 | $46.12^0$ | 0.7208 | |
| 0.96 | $43.83^0$ | 0.6925 | *number of data ³ 3* |
| 1.08 | $47.20^0$ | 0.7330 | [***0.5 pts***] |

| Name of component no.2 | Specification | |
|---|---|---|
| Diffraction grating | Spacing = 0.936 μm | **[*0.4 pts*]** |
| | Lines parallel to the slit | **[*0.1 pts*]** |

Note: true value of grating spacing is 1.0 μm, deviation of the result ≤ 10%

6. Plan-parallel plate **[total 2.0 pts]**:




*drawing and labels should be complete*

**[*0.4 pts*]**

Snell's law:

$\sin\varphi = n \sin\varphi'$ , $\varphi' = \angle BAC$

Path length inside the plate:

$AC = AB / \cos\varphi'$ , $AB = h =$ plate thickness

Beam displacement:

$CD = t = AC \sin\angle CAD$ , $\angle CAD = \varphi - \varphi'$

Hence:

$t = h \sin\varphi \left[ 1 - \cos\varphi / (n^2 - \sin^2\varphi)^{1/2} \right]$ [***0.6 pts***]

<u>Observation data:</u>

| $\varphi$ | $t$ | |
|---|---|---|
| 0 | 0 | (angle between beam and axis 49°) |
| $49^0$ | 7.3 arbitrary scale | [***0.5 pts***] |

| Name of component no.3 | Specification | |
|---|---|---|
| Plane-parallel plate | Thickness = 17.9 mm | **[*0.2 pts*]** |
| | Angle to the axis of the box 49° | **[*0.3 pts*]** |

Note: - true value of plate thickness is 20 mm
- true value of angle to the axis of the box is 52°
- deviation of the results ≤ 20%.