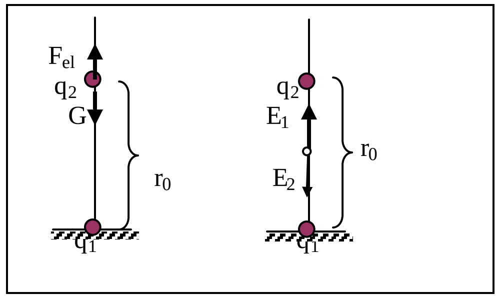
MINISTERUL EDUCATIEI NATIONALE
INSPECTORATUL SCOLAR JUDETEAN GALATI
CONCURSUL NATIONAL EVRIKA 2000

CLASA a IX-a - REZOLVARI



Problema 1

1) $\vec{F}_{el} + \vec{G} = 0$

$$F_{el} = mg \quad \Leftrightarrow \quad \frac{kq_1q_2}{r_0^2} = mg$$

$$r = \sqrt{\frac{kq_1q_2}{mg}} = 15.10^{-2}\,m = 15\,cm \qquad (\textit{1 punct})$$

2) $\vec{E} = \vec{E}_1 + \vec{E}_2$, $E = |E_1 - E_2| = \left| k\frac{4q_1}{r_0^2} - k\frac{4q_2}{r_0^2} \right| = \frac{4k}{r_0^2}|q_1 - q_2| = 3{,}84.10^6\ \frac{V}{m}$

(*1,5 puncte*)

3) Sfera atinge viteza maxima cand $\vec{F} + \vec{G} = 0$ adica cand trece prin punctul aflat la r_0 = 15 cm de sfera fixa.

$$\Delta E_c = L_{el} + L_g$$

$$\frac{mv_{max}^2}{2} - 0 = q_2\left(\frac{kq_1}{r_1} - \frac{kq_2}{r_2}\right) - mg(r_0 - r_1) = 70{,}7\,cm/s \qquad (\textit{2,5 puncte})$$

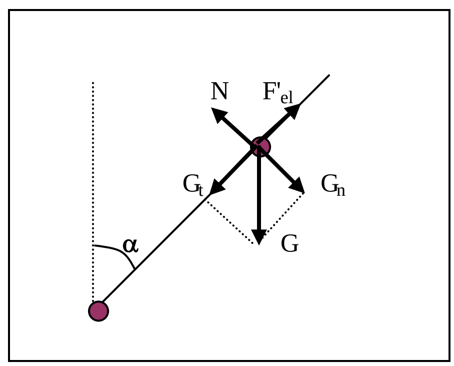
$$\Delta E_c' = L' \quad \Leftrightarrow \quad 0 = L_{el}' + L_g' \quad \Leftrightarrow \quad L_{el}' = -L_g'$$

4) $Dar\ -L_g' = \Delta E_{pg} \quad \Leftrightarrow \quad \Delta E_{pg} = L_{el}'$

$$mg(r_2 - r_1) = q_2\left(\frac{kq_1}{r_1} - \frac{kq_2}{r_2}\right)$$

de unde r_2 = 22,5 cm (*2 puncte*)

Sfera va oscila intre punctele aflate la distantele r_1 = 10 cm si r_2 = 22,5 cm (*0,5 puncte*)



5) $G_t = F_{el}'$ $\qquad mg\cos\alpha = \frac{kq_1q_2}{r_0'^2}$

(1 punct)

$$r'_0 = \sqrt{\frac{kq_1q_2}{mg\cos\alpha}} = 21{,}15\,cm \qquad (\textit{1,5 puncte})$$

Problema 2

Puterea electrica transferata rezistorului avand rezistenta electrica R = nr, are valoarea

$$P = RI^2 = \frac{RE^2}{(r+R)^2} = \frac{nE^2}{r(1+n)} \qquad (1) \qquad (\textit{2 puncte})$$

Puterea electrica maxima ce poate fi transferata rezistorului din circuitul exterior (n = k = 1) este

$$P_{max} = \frac{E^2}{4r} \qquad (2) \qquad (\textit{2 puncte})$$

Punand conditia $P = kP_{max}$, din (1) si (2) rezulta $kn^2 - 2(2-k)n + k = 0$ (3) (*2 puncte*)

Rezolvand ecuatia (3)in raport cu n, avem

$$\mathbf{n}_{1,2} = \frac{2}{\mathbf{k}}\left(1 \pm \sqrt{1-\mathbf{k}}\right) - 1 \qquad (4)$$

substituind k = 8/9 in (4) se obtin $n_1 = 2$ si $n_2 = 1/2$ (***3 puncte***)

Problema 3

A. 1) Abscisa si ordonata lui M sunt date de relatiile $x = mt^2$ si $y = -nt$ (***0,5 puncte***)

Din a doua relatie avem $\mathbf{t} = -\frac{\mathbf{y}}{\mathbf{n}}$. Rezulta ecuatia traiectoriei $\mathbf{x} = \frac{\mathbf{m}}{\mathbf{n}^2}\mathbf{y}^2$ (***0,5 puncte***)

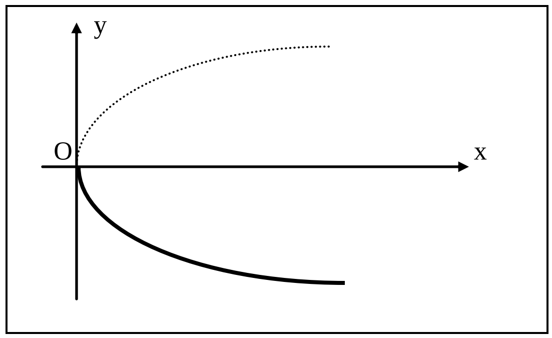
Graficul traiectoriei este un arc de parabola ca in figura.

2) Componentele vitezei pe cele doua axe sunt

$\mathbf{v_x} = 2\mathbf{mt}$ (0,25 puncte) si $\mathbf{v_y} = -\mathbf{n}$ (***0,25 puncte***)

Componentele acceleratiei pe cele doua axe : $\mathbf{a_x} = 2\mathbf{m}$ (***0,25 puncte***) si

$\mathbf{a_y} = 0$ (0,25 puncte)



Vectorii viteza si acceleratie instantanee:

$\vec{\mathbf{v}} = \mathbf{v_x}\vec{\mathbf{i}} + \mathbf{v_y}\vec{\mathbf{j}} = 2\mathbf{mt}\vec{\mathbf{i}} - \mathbf{n}\vec{\mathbf{j}}$ (***0,5 puncte***)

$\vec{\mathbf{a}} = \mathbf{a_x}\vec{\mathbf{i}} + \mathbf{a_y}\vec{\mathbf{j}} = 2\mathbf{m}\vec{\mathbf{i}}$ (***0,5 puncte***)

cu modulele $\mathbf{v} = \sqrt{4\mathbf{m}^2\mathbf{t}^2 + \mathbf{n}^2}$ (***0,25 puncte***)

$\mathbf{a} = 2\mathbf{m}$ (***0,25 puncte***)

3) $\mathbf{v}_{\mathbf{med_x}}(\tau) = \frac{\Delta\mathbf{x}}{\tau} = \mathbf{m}\tau$ (***0,25 puncte***)

$\mathbf{v}_{\mathbf{med_y}}(\tau) = \frac{\Delta\mathbf{y}}{\tau} = -\mathbf{n}$ (***0,25 puncte***) desen (***0,5 puncte***)

$\vec{\mathbf{v}}_{\mathbf{med}} = \mathbf{m}\tau\vec{\mathbf{i}} - \mathbf{n}\vec{\mathbf{j}}$ (***0,25 puncte***)

$\mathbf{v}_{\mathbf{med}} = \sqrt{\mathbf{m}^2\tau^2 + \mathbf{n}^2}$ (***0,25 puncte***)

B. In momentul intalnirii trenurilor se incruciseaza si fumurile lor in punctul P_0 situat la jumatatea distantei dintre trenuri. Fumul se misca solidar cu aerul din P_0 in P cu viteza v. Avem:

$\mathbf{t} = \frac{2\mathbf{x}}{\mathbf{v}} = 20\mathbf{s}$; $|\mathbf{v_x}| = \frac{\mathbf{x}}{\mathbf{t}} = 5\mathbf{m/s}$ (***1 punct***)

$\mathbf{v_y} = \frac{2\mathbf{x}}{\mathbf{t}} = 10\mathbf{m/s}$ (***0,5 puncte***)

$\mathbf{v} = \sqrt{\mathbf{v_x^2} + \mathbf{v_y^2}} = 11{,}18\mathbf{m/s}$ (***1 punct***)

$\mathbf{tg}\alpha = \frac{\mathbf{v_y}}{|\mathbf{v_x}|} = 2$; $\alpha = 63^0 26'$ (***1 punct***) desen (***0,5 puncte***)