27^{th} INTERNATIONAL PHYSICS OLYMPIAD
OSLO, NORWAY

THEORETICAL COMPETITION
JULY 2 1996

Time available: 5 hours

READ THIS FIRST :

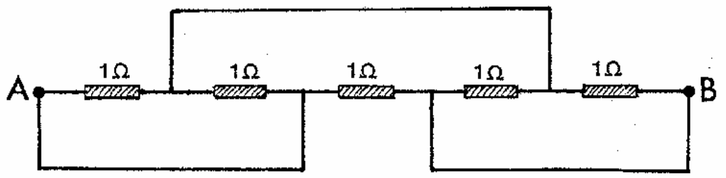
1. Use only the pen provided
2. Use only the marked side of the paper
3. Each problem should be answered on separate sheets
4. In your answers please use primarily equations and numbers, and as little *text* as possible
5. Write at the top of *every* sheet in your report:
    - Your candidate number (IPhO identification number)
    - The problem number and section identification, e.g. 2/a
    - Number each sheet consecutively
6. Write on the front page the total number of sheets in your report

This set of problems consists of 7 pages.

PROBLEM 1

(The five parts of this problem are unrelated)

a) Five 1Ω resistances are connected as shown in the figure. The resistance in the conducting wires (fully drawn lines) is negligible.



Determine the resulting resistance R between A and B. *(1 point)*

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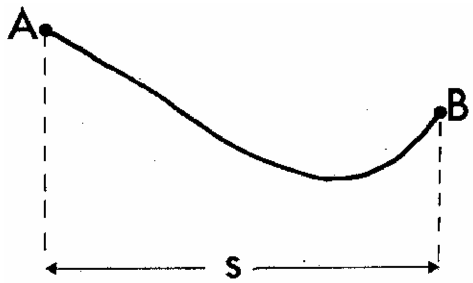
b)



A skier starts from rest at point A and slides down the hill, without turning or braking. The friction coefficient is μ. When he stops at point B, his horizontal displacement is s. What is the height difference h between points A and B. (The velocity of the skier is small so that the additional pressure on the snow due to the curvature can be neglected. Neglect also the friction of air and the dependence of μ on the velocity of the skier.) *(1.5 points)*

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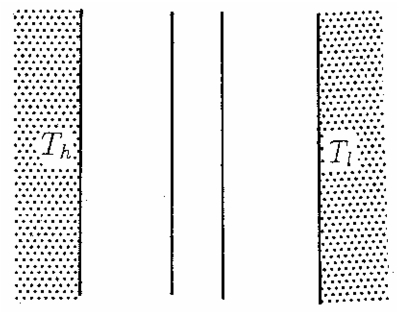
c) A thermally insulated piece of metal is heated under atmospheric pressure by an electric current so that it receives electric energy at a constant power P. This leads to an increase of the absolute temperature T of the metal with time t as follows:

$$T(t) = T_0\left[1 + a(t - t_0)\right]^{1/4}.$$

Here a, t_0 and T_0 are constants. Determine the heat capacity $C_p(T)$ of the metal (temperature dependent in the temperature range of the experiment). *(2 points)*

d) A black plane surface at a constant high temperature T_h is parallel to another black plane surface at a constant lower temperature T_l. Between the plates is vacuum.

In order to reduce the heat flow due to radiation, a heat shield consisting of two thin black plates, thermally isolated from each other, is placed between the warm and the cold surfaces and parallel to these. After some time stationary conditions are obtained.



By what factor ξ is the stationary heat flow reduced due to the presence of the heat shield? Neglect end effects due to the finite size of the surfaces. *(1.5 points)*

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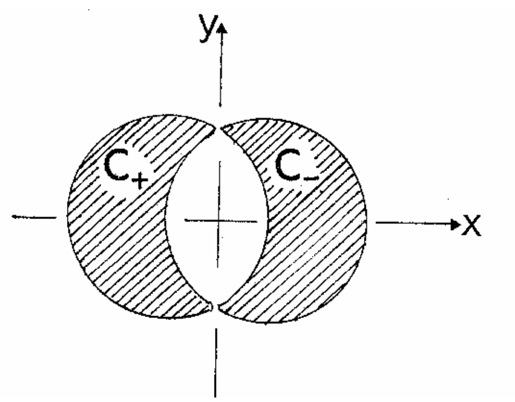
e) Two straight and very long nonmagnetic conductors C_+ and C_-, insulated from each other, carry a current I in the positive and the negative z direction, respectively. The cross sections of the conductors (hatched in the figure) are limited by circles of diameter D in the x-y plane, with a distance $D/2$ between the centres. Thereby the resulting cross sections each have an area $(\frac{1}{12}\pi + \frac{1}{8}\sqrt{3})D^2$. The current in each conductor is uniformly distributed over the cross section.

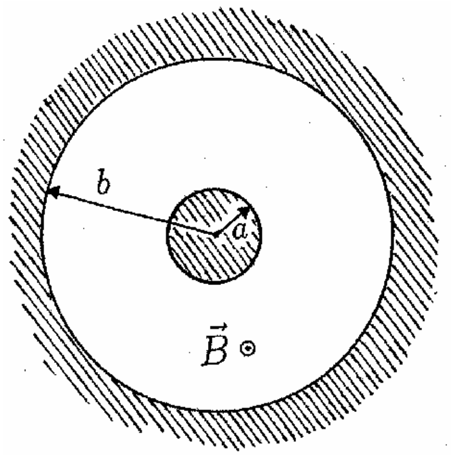


Determine the magnetic field $B(x,y)$ in the space between the conductors. *(4 points)*

PROBLEM 2

The space between a pair of coaxial cylindrical conductors is evacuated. The radius of the inner cylinder is a, and the inner radius of the outer cylinder is b, as shown in the figure below. The outer cylinder, called the anode, may be given a positive potential V relative to the inner cylinder. A static homogeneous magnetic field $\vec{B}$ parallel to the cylinder axis, directed out of the plane of the figure, is also present. Induced charges in the conductors are neglected.

We study the dynamics of electrons with rest mass m and charge $-e$. The electrons are released at the surface of the inner cylinder.



a) First the potential V is turned on, but $\vec{B} = 0$. An electron is set free with negligible velocity at the surface of the inner cylinder. Determine its speed v when it hits the anode. Give the answer both when a non-relativistic treatment is sufficient, and when it is not. *(1 point)*

For the remaining parts of this problem a non-relativistic treatment suffices.

b) Now $V = 0$, but the homogeneous magnetic field $\vec{B}$ is present. An electron starts out with an initial velocity $\vec{v}_0$ in the radial direction. For magnetic fields larger than a critical value B_c, the electron will not reach the anode. Make a sketch of the trajectory of the electron when B is slightly more than B_c. Determine B_c. *(2 points)*

From now on *both* the potential V *and* the homogeneous magnetic field $\vec{B}$ are present.

c) The magnetic field will give the electron a non-zero angular momentum L with respect to the cylinder axis. Write down an equation for the rate of change *dL/dt* of the angular momentum. Show that this equation implies that

$$L - keBr^2$$

is constant during the motion, where k is a definite pure number. Here r is the distance from the cylinder axis. Determine the value of k. *(3 points)*

d) Consider an electron, released from the inner cylinder with negligible velocity, that does not reach the anode, but has a maximal distance from the cylinder axis equal to r_m. Determine the speed v at the point where the radial distance is maximal, in terms of r_m. *(1 point)*

e) We are interested in using the magnetic field to regulate the electron current to the anode. For B larger than a critical magnetic field B_c, an electron, released with negligible velocity, will not reach the anode. Determine B_c. *(1 point)*

f) If the electrons are set free by heating the inner cylinder an electron will in general have an initial nonzero velocity at the surface of the inner cylinder. The component of the initial velocity parallel to $\vec{B}$ is v_B, the components orthogonal to $\vec{B}$ are v_r (in the radial direction) and v_φ (in the azimuthal direction, i.e. orthogonal to the radial direction).

Determine for this situation the critical magnetic field B_c for reaching the anode. *(2 points)*

PROBLEM 3

In this problem we consider some gross features of the magnitude of mid-ocean tides on earth. We simplify the problem by making the following assumptions:

(i) The earth and the moon are considered to be an isolated system,
(ii) the distance between the moon and the earth is assumed to be constant,
(iii) the earth is assumed to be completely covered by an ocean,
(iv) the dynamic effects of the rotation of the earth around its axis are neglected, and
(v) the gravitational attraction of the earth can be determined as if all mass were concentrated at the centre of the earth.

The following data are given:

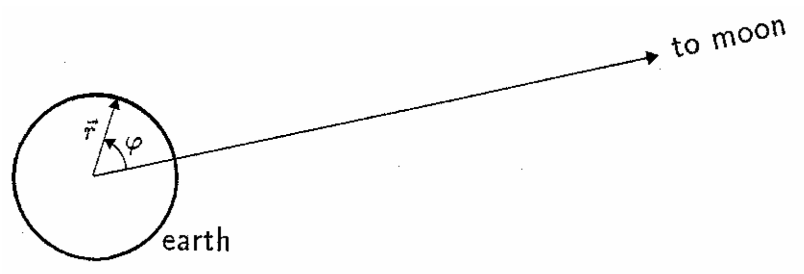
Mass of the earth: $M = 5.98 \cdot 10^{24}$ kg
Mass of the moon: $M_m = 7.3 \cdot 10^{22}$ kg
Radius of the earth: $R = 6.37 \cdot 10^{6}$ m
Distance between centre of the earth and centre of the moon: $L = 3.84 \cdot 10^{8}$ m
The gravitational constant: $G = 6.67 \cdot 10^{-11}\,\mathrm{m^3kg^{-1}s^{-2}}$.

a) The moon and the earth rotate with angular velocity ω about their common centre of mass, C. How far is C from the centre of the earth? (Denote this distance by l.)

Determine the numerical value of ω. *(2 points)*

We now use a frame of reference that is co-rotating with the moon and the center of the earth around C. In this frame of reference the shape of the liquid surface of the earth is static.

In the plane P through C and orthogonal to the axis of rotation the position of a point mass on the liquid surface of the earth can be described by polar coordinates r, φ as shown in the figure. Here r is the distance from the centre of the earth.



We will study the shape

$$r(\varphi) = R + h(\varphi)$$

of the liquid surface of the earth in the plane P.

b) Consider a mass point (mass m) on the liquid surface of the earth (in the plane P). In our frame of reference it is acted upon by a centrifugal force and by gravitational forces from the moon and the earth. Write down an expression for the potential energy corresponding to these three forces.

Note: Any force $F(r)$, radially directed with respect to some origin, is the negative derivative of a spherically symmetric potential energy $V(r)$: $F(r) = -V'(r)$. *(3 points)*

c) Find, in terms of the given quantities M, M_m, etc, the approximate form $h(\varphi)$ of the tidal bulge. What is the difference in meters between high tide and low tide in this model?

You may use the approximate expression

$$\frac{1}{\sqrt{1+a^2-2a\cos\theta}} \approx 1 + a\cos\theta + \tfrac{1}{2}a^2(3\cos^2\theta - 1),$$

valid for a much less than unity.

In this analysis make simplifying approximations whenever they are reasonable. *(5 points)*